

SG51 GEOMETRY PROBLEMS

2nd ROUND

by February 15, 2017

You may take as many breaks as you like, but they may add up to no more than **1 month**.

Reference policy: No interaction with your fellow students, teachers or parents.

This is an INDIVIDUAL competition.

Grading:

5 points: clear, concise, correct solution

4 points: minor shortcoming in solution

3 points: larger holes in a promising solution

2 points: some good ideas but not much more

1 point: evidence of understanding the problem

0 points: nothing presented that is credible

You should submit FULL DETAILS of the working of your solutions and of your particulars (Full name, home address, school and class and email).

Send your solutions to:

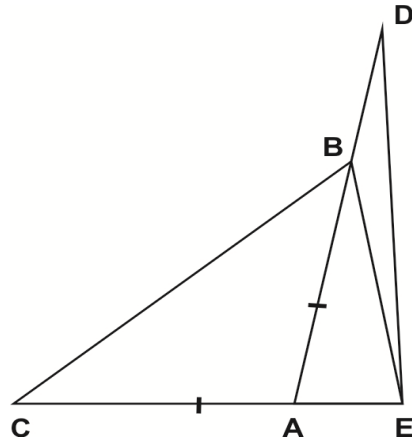
williey2014@gmail.com

and

mwb_en@mathematicalmail.com

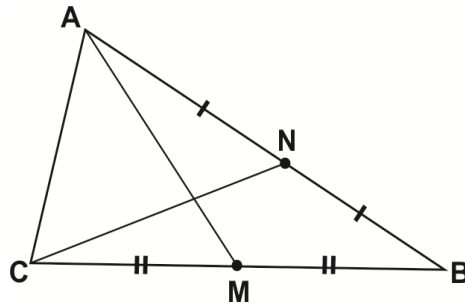
Prizes will be awarded to the top 10 contestants.

1.



Given $AB = AC$,
 $DA = DE = CE =$
 CB .
 Find $\sphericalangle BAC$.

2.

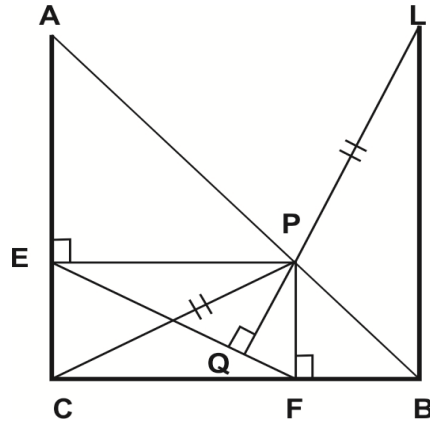


In $\triangle ABC$, $AB = 4$,
 $BC = 3$
 and $AC = \sqrt{3}$.
 M, N are the
 midpoints of BC and
 AB respectively.
 Show that $AM \perp CN$!

3.

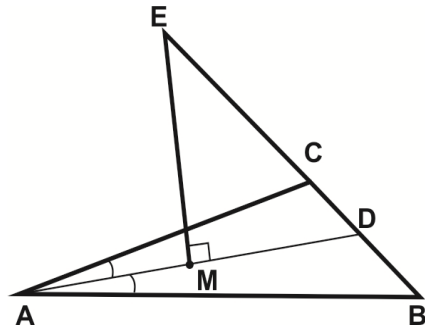
The diagonals of
 quadrilateral $ABCD$
 intersect at O .
 $\sphericalangle BAC = 25^\circ$,
 $\sphericalangle BCA = 20^\circ$,
 $\sphericalangle CDB = 50^\circ$ and
 $\sphericalangle ADC = 90^\circ$.
 Find $\sphericalangle COD$.

4.



In $\triangle ABC$, $\sphericalangle C = 90^\circ$,
 $CA = CB$.
 $PE \perp AC, PF \perp BC$
 and $PQ \perp EF$.
 QPL is a straight
 line,
 $PL = PC$.
 Show that $LB \perp BC$!

5.

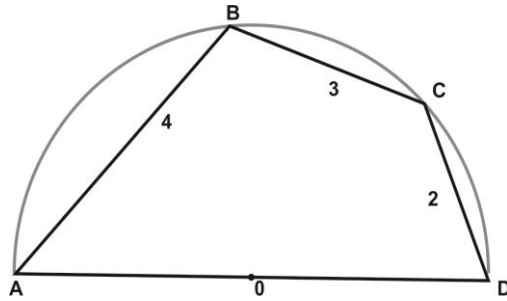


M is a midpoint of
 AD . AD bisects
 $\sphericalangle CAB$. BC is
 extended to E such
 that $\sphericalangle DME = 90^\circ$.
 Show that
 $DE^2 = BE \times CE$.

6.

P is any point inside
 $\triangle ABC$.
 $\sphericalangle PBA = \sphericalangle PCA$.
 PE and PF are
 perpendicular to AB
 and AC respectively.
 D is the midpoint of
 BC .
 Show that
 $DE = DF$.

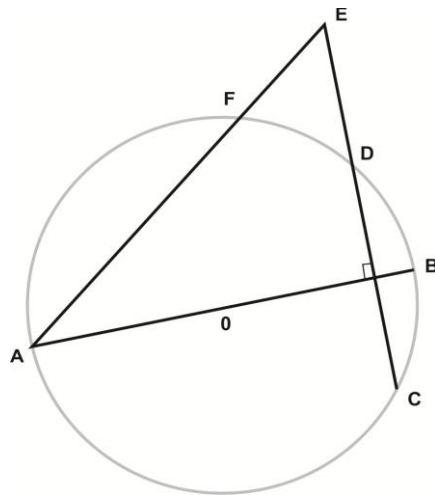
BONUS QUESTION 1



In quadrilateral $ABCD$,
 $AB = 4$, $BC = 3$ and $CD = 2$.
 Find the diameter, AD .

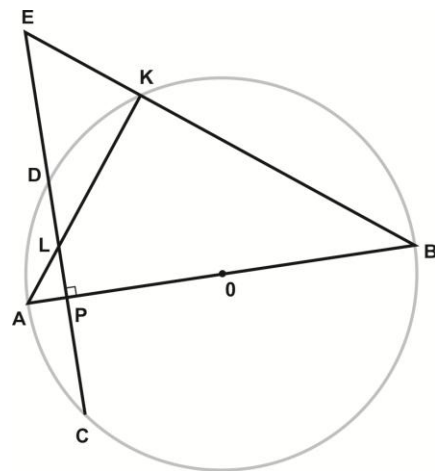
BONUS QUESTION 1

a)



$AB \perp CD$. CDE & AFE are
 straight lines.
 $AC \times FD = AF \times DE!$

b)



$CP^2 = PL \times PE!!$
 ALK and EKB are straight
 lines.