SG51 GEOMETRY PROBLEMS

5th ROUND

by 31 May, 2017

You may take as many breaks as you like, but they may add up to no more than **1 month.**

Reference policy: No interaction with your fellow students, teachers or parents.

This is an INDIVIDUAL competition.

Grading:

5 points: clear, concise, correct solution

4 points: minor shortcoming in solution

3 points: larger holes in a promising solution

2 points: some good ideas but not much more

1 point: evidence of understanding the problem

0 points: nothing presented that is credible

You should submit FULL DATAILS of the working of your solutions and of your particulars (Full name, home address, school and class and email).

Send your solutions to:

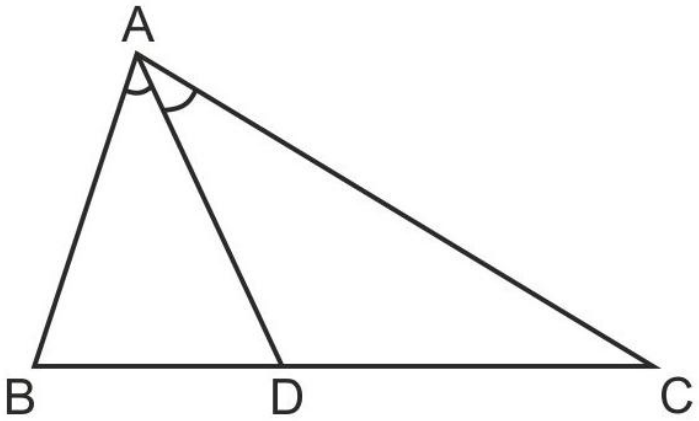
williey2014@gmail.com

and

mwb_en@mathematicalmail.com

Prizes will be awarded to the top 10 contestants.

1.



CA = CB, CD = AB.
AD bisects $\sphericalangle$ *BAC.*
Find $\sphericalangle$ *ACB.*

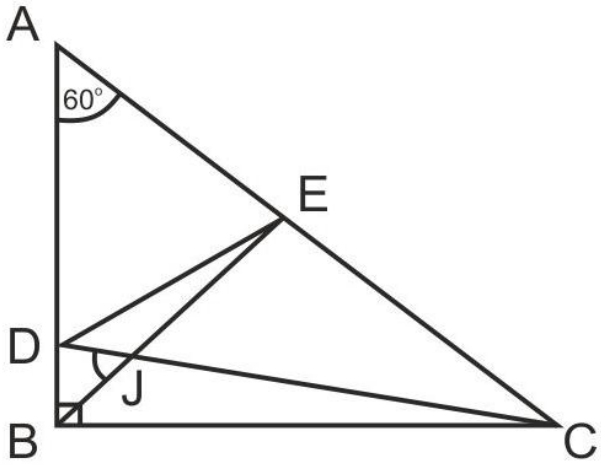
2.

AB = AC. M, N are midpoints of *AB, AC* respectively. *G* is the centroid of ΔAMC. *O* is the centre of circle *ABC.*
Show that $GO \perp MC$.

3.

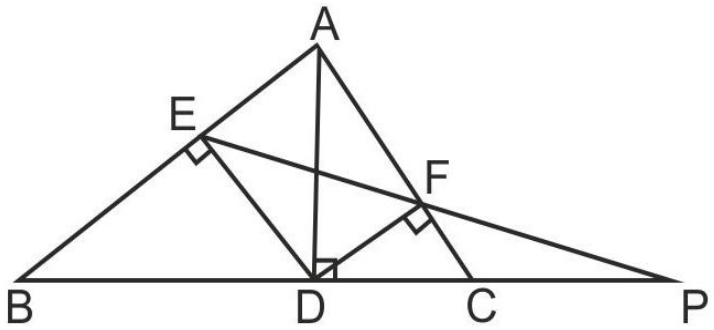
AB is a diameter. Chord *CD* is perpendicular to *AB.*
M is the midpoint of *OC.*
AM meets the circle at *E.*
DE intersects *BC* at *P.*
Show that *CP = PB.*

4.



AD = AE.
$\sphericalangle$ *DAE* = $\sphericalangle$ *DJB* = 60^0.
Find *EJ : JD.*
All lines are straight.

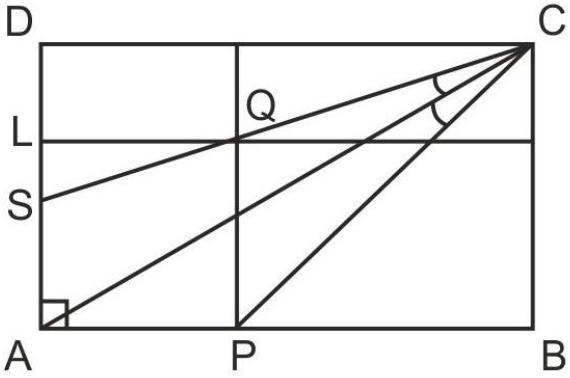
5.



$AD \perp BC$, $DE \perp AB$ and $DF \perp AC$.
$EF \cap BC$ at *P.*

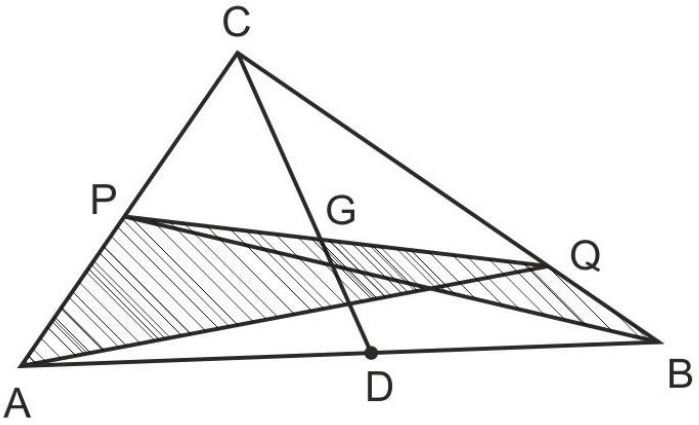
Show that $\frac{1}{DC} = \frac{1}{DP} + \frac{1}{BD}$.

6.



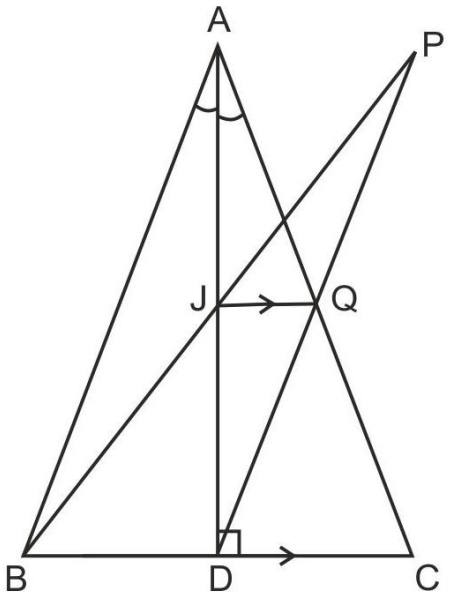
ABCD is a rectangle.
APQL is a square.
AC bisects $\sphericalangle SCP$.
$\sphericalangle CAP = \sphericalangle PCQ$.
Find $\sphericalangle CSA$.

7.



CD is a median and *G* is the centroid of ΔABC.
PGQ is a straight line.
Show that
$[AQP] + [BQP] \geq \frac{4}{9}[ABC]$.

BONUS QUESTION



AD bisects $\sphericalangle BAC$
and $\sphericalangle ADC = 90^0$.
$JQ \parallel BC$.
$BJ \cap DQ$ at *P*.
Show that $AP \parallel JQ$.