SG51 GEOMETRY PROBLEMS

6th ROUND

By 30 August, 2017

You may take as many breaks as you like, but they may add up to no more than **1 month.**

Reference policy: No interaction with your fellow students, teachers or parents.

This is an INDIVIDUAL competition.

Grading:

5 points: clear, concise, correct solution

4 points: minor shortcoming in solution

3 points: larger holes in a promising solution

2 points: some good ideas but not much more

1 point: evidence of understanding the problem

0 points: nothing presented that is credible

You should submit FULL DATAILS of the working of your solutions and of your particulars (Full name, home address, school and class and email).

Send your solutions to:

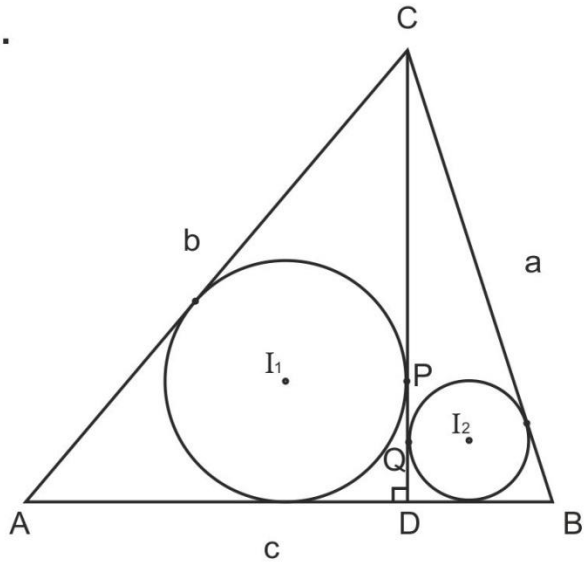
williey2014@gmail.com

and

mwb_en@mathematicalmail.com

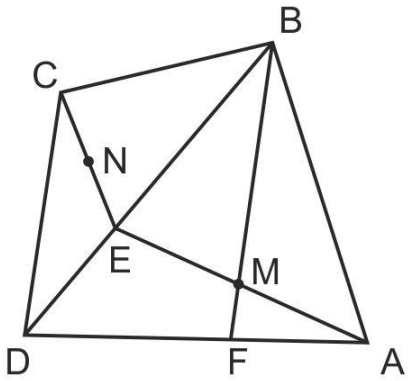
Prizes will be awarded to the top 10 contestants.

1.



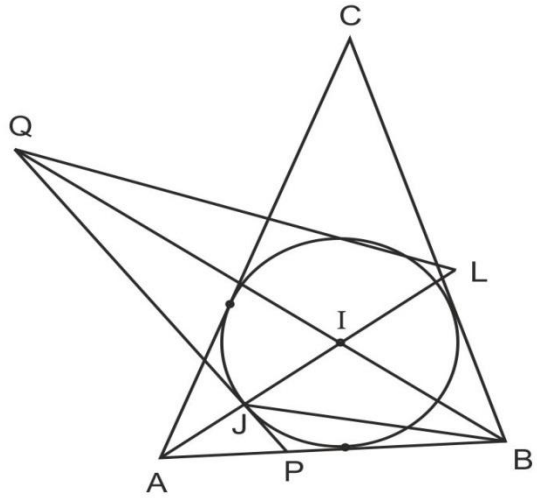
Find *PQ* in terms of *a*, *b* and *c*.

2.



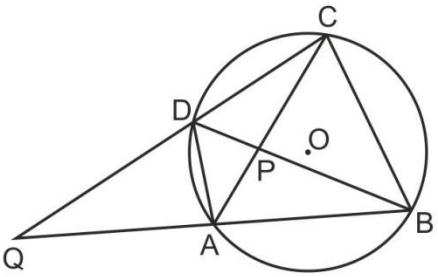
CNE, *DEB*, *BMF* and *EMA* are straight lines.
M, *N* are the midpoints of *AE* and *CE* respectively.
If *DE:DB = m* and *CN:CE = p*, find *BM:BF*.

3.



BI is extemded to *Q*.
QP touches incircle at *J*.
AI is extended to *L* such that *QL* ∥ *JB*.
Prove that *CL* ⊥ *AL*.

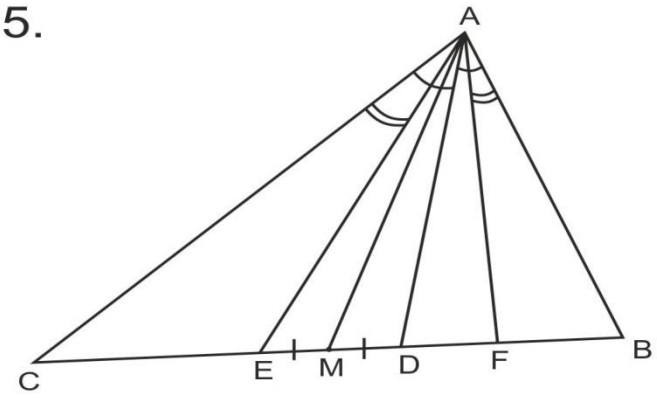
4.



$CD \cap BA$ at Q.

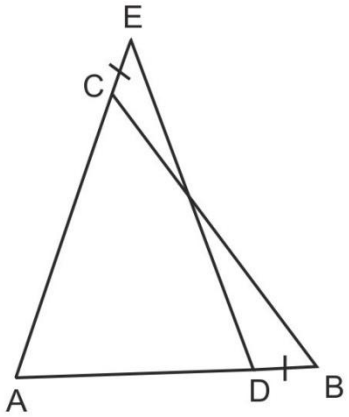
What's your conjecture?

5.



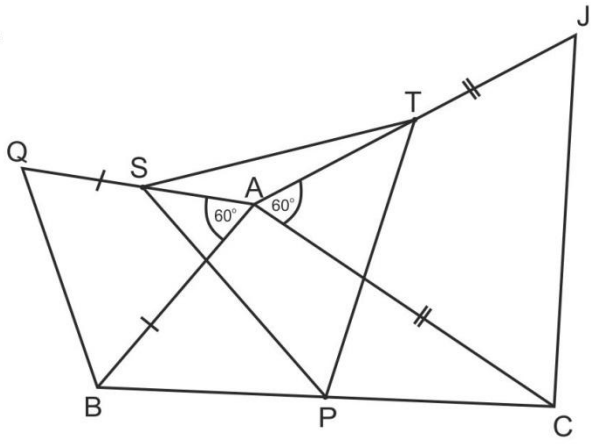
In ΔABC, AM is a median, AD bisects $\sphericalangle CAB$, $\sphericalangle CAE = \sphericalangle BAF$ and $ME = MD$.
Find $BF:FC$ in terms of b and c.

6.



$AB = AC$, $CE = BD$.
Show that $DE > BC$!
Seeing is not quite BELEIVING!!

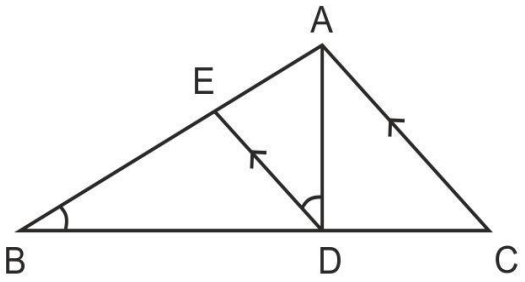
7.



ABQ and *ACJ* are equilateral triangles.
P, *S* and *T* are the midpoints of *BC*, *AQ* and *AJ* respectively.

PST is an equilateral triangle!

8.



In ΔABC, $\measuredangle ADE = \measuredangle ABE$ and $DE \parallel CA$.
Let the perimeters of triangles *ABC*, *DAC* and *EBD* be *p*, p_1 and p_2 respectively.

Show that $\frac{p_1 + p_2}{p} \leq \frac{5}{4}$.