

SG51 GEOMETRY PROBLEMS

7th ROUND

By 30 August, 2017

You may take as many breaks as you like, but they may add up to no more than **1 month**.

Reference policy: No interaction with your fellow students, teachers or parents.

This is an INDIVIDUAL competition.

Grading:

5 points: clear, concise, correct solution

4 points: minor shortcoming in solution

3 points: larger holes in a promising solution

2 points: some good ideas but not much more

1 point: evidence of understanding the problem

0 points: nothing presented that is credible

You should submit FULL DETAILS of the working of your solutions and of your particulars (Full name, home address, school and class and email).

Send your solutions to:

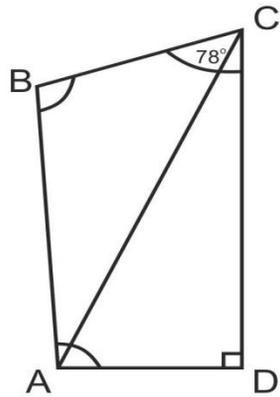
williey2014@gmail.com

and

mwb_en@mathematicalmail.com

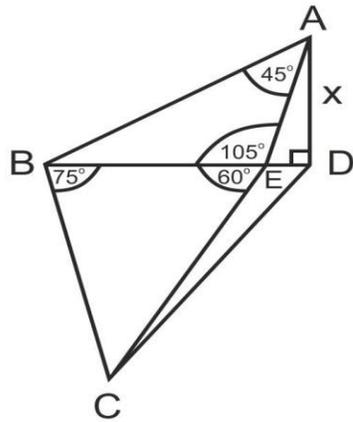
Prizes will be awarded to the top 10 contestants.

1.



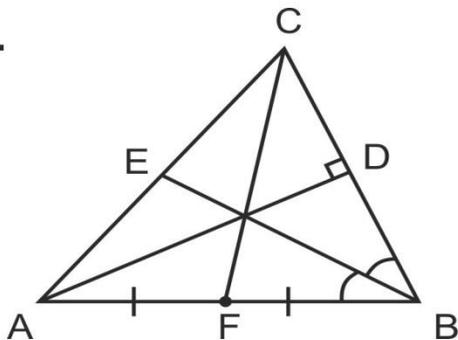
In quadrilateral $ABCD$,
 $\angle BCD = 78^\circ$, $\angle ADC = 90^\circ$.
 $\angle DAB = \angle ABC$ and
 $AB = 2AD$.
 Find $\angle DAC$.

2.



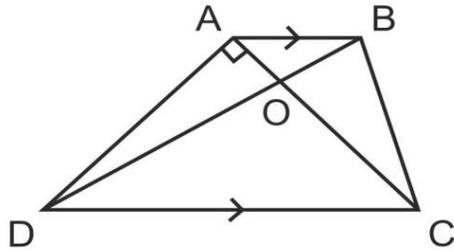
Given $AD = x$,
 Find CE in terms of x .

3.



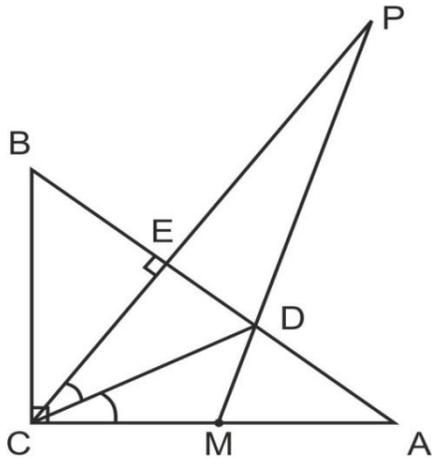
AD is an altitude, BE bisects
 $\angle ABC$ and CF is a median.
 Find an equation connecting a , b
 and c .
 The lines AB , BE and CF are
 concurrent.

4.



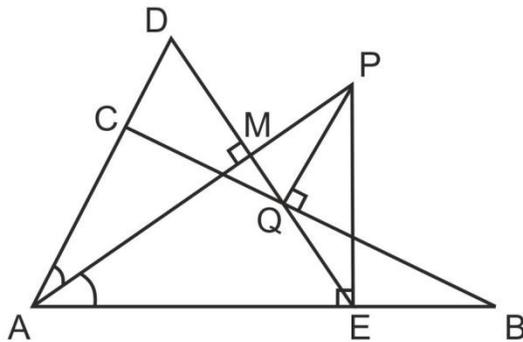
$AB \parallel DC$
 $\angle CAD = 90^\circ$.
 $AC = AD, DB = DC$.
 Show that $CO = BO$!

5.



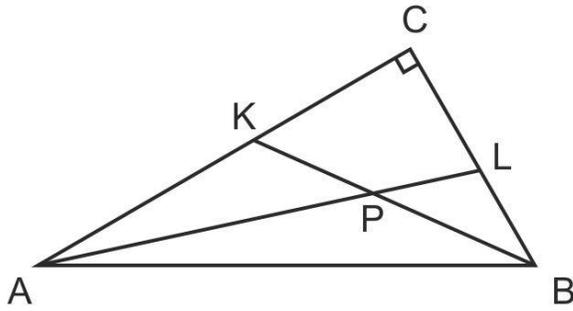
CD bisects $\angle C$ ($\angle C = 90^\circ$).
 M is the midpoint of AC .
 CE is perpendicular to AB .
 CD bisects $\angle ECA$.
 $CE \cap MD$ at P .
 Prove: $BP \parallel CD$.

6.



AC is extended to D so that
 $AE = AD$.
 AP bisects $\angle CAB, AP \perp DE$.
 PE is perpendicular to BC .
 M is the midpoint of DE .
 Prove: $QC = QB$.
 Note that AMP, CQB and DQE
 are straight lines.

Bonus Question

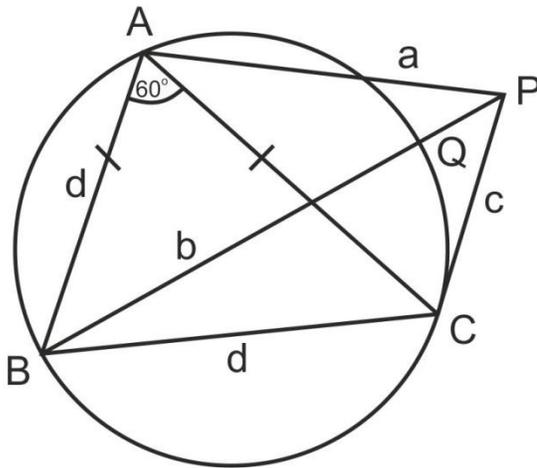


In $\triangle ABC$, $\sphericalangle C = 90^\circ$ and $AC > BC$, $AK = BC$.

a) If $BL = KC$, show that $\sphericalangle APK = 45^\circ$.

b) If $\sphericalangle APK = 45^\circ$, show that $BL = KC$.

Bonus Question



ABC is an equilateral triangle.

P is an external point.

PB cuts the circle at Q .

If $PA = a$, $PB = b$, $PC = c$ and $AB = d$, find a formula connecting a , b , c and d .