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MATHEMATICS WITHOUT BORDERS

AGE GROUP 1

INSTRUCTIONS

1. Please DO NOT OPEN the contest papers until the Exams Officer has given permission.

2. There are 20 open-ended questions in the test. Please write down your answers in the ANSWER SHEET.

3. Each problem is scored 2 points for a correct answer; 1 point if there are two or more answers and at least half are given, or if one wrong answer is given in addition to the correct answer; 0 points for a wrong answer or no answer.

4. Working time: not more than 60 minutes. In the case of an equal number of points, the higher ranked participant will be the one who has spent less time solving the problems.

5. The use of calculators or other electronic devices, as well as books containing formulae is NOT allowed during the course of the contest.

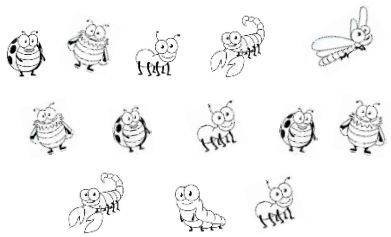
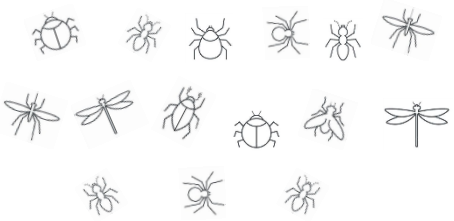
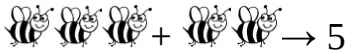
6. The problems contain natural numbers, as well as 0. (0, 1, 2, 3, 4, ..., 10, 11, ..., 99, 100, 101, ...)

7. For the problems containing a numerical answer, natural numbers, as well as 0 must be used. (0, 1, 2, 3, 4, ..., 10, 11, ..., 99, 100, 101, ...)

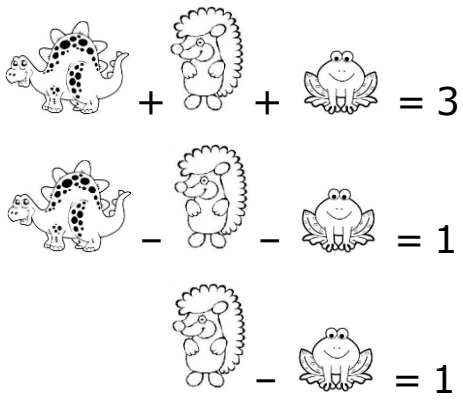
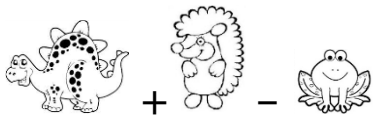
8. Students are NOT allowed to receive help by the Exams Officer or by anyone else during the contest.

WE WISH YOU ALL SUCCESS!

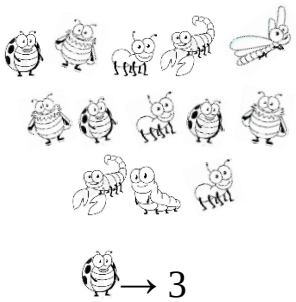
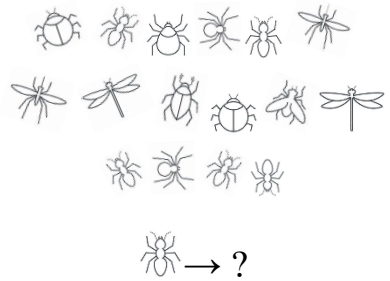
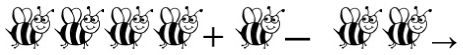
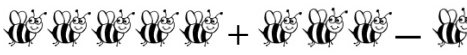
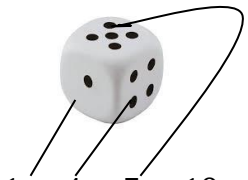
AUTUMN – AGE GROUP 1

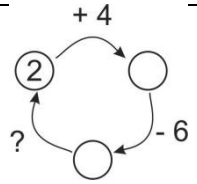
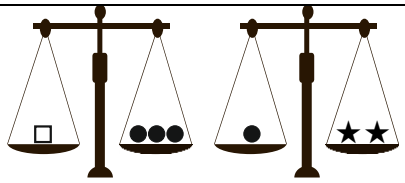
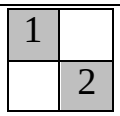
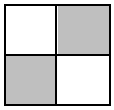
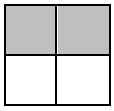
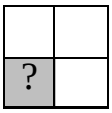
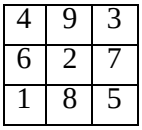
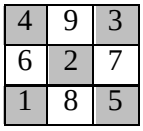
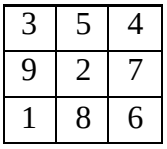
	 **EXAMINE**	 **THINK**	 **WRITE**
1	 → 3	 → ?	…..
2	1, 2, 3 → 1 + 2 + 3 = 6 ? = 6	2, 3, 4 → …+…+…= ? ? =	…..
3	 → 5	 → ?	…..
4	$\underbrace{1,2,\ldots,\ldots,5,6}_{3+4=7}$? = 7	$\underbrace{2,3,\ldots,\ldots,6,7}_{\ldots+\ldots=?}$? =	…..
5	●1●●2●●●? ? = 3	●1●●3 ●●●?●●●●7●●●●●9 ? =	…..
6	1 → 3 → ? → 7 → 9 ? = 5	0 → 2 → 4 → 6 → ? → 10 ? =	.....
7	 ? = 5 + 1 ? = 6	 … +… + … = ? ? =	…..
8	3 – 2 + 4 – 3 + → – – → + 3 + 2 – 4 + 3 = ? ? = 4	3 + 2 – 1 – 3 + → – – → + 3…2…1…3 = ? ? =	…..

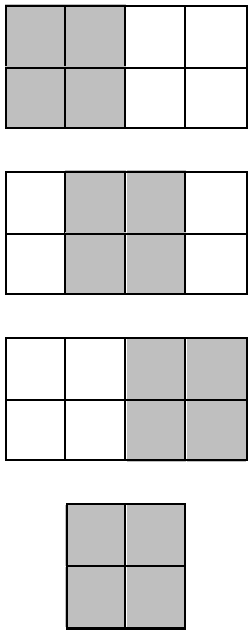
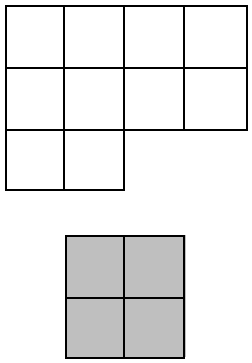
9	$3 + 2 < 3 + @$ $6 - 2 > @$ $@ = 3$	$3 + 2 > 3 + @$ $2 - 2 < @$ $@ = ?$	.....
10	$1 + 1 + 1 + 1 = 4$ $1 \rightarrow 2$ $2 + 2 + 2 + 2 = ?$ $? = 8$	$2 + 2 + 2 = 6$ $2 \rightarrow 3$ $\ldots + \ldots + \ldots + = ?$ $? =$	.....
11	12345678 1357 15 $? = 15$	11223344 .... $? =$	.....
12	★ + ★ = 4 ★ + ★ + ★ + ★ = ? $? = 8$	✹ + ✹ = 2 ✹ + ✹ + ✹ + ✹ + ✹ + ✹=? $? =$	.....
13	★ + ★ = 2 ★ + ? = 1 $? = 0$	★ + ★ = 4 ★ + ? = 3 $? =$	.....
14	1, 2, 3, 5 $1 + 2 + 3 - 5 = ?$ $? = 1$	2, 3, 5, 1 $\ldots + \ldots + \ldots - \ldots = ?$ $? =$	.....
15	$\underbrace{7,8,9}_{3}$ $\underbrace{10}_{2}$ $\underbrace{7,8,9,10}_{5}$	$\underbrace{6,7,8,9,10}_{?}$ $? =$	.....
16	✻ ✻ = ❀ ❀ ❀ ❀ ❀ ❀ ❀ ❀ ❀ = 4(✻) $? = 4$	✻ ✻ ✻ = ❀ ❀ ❀ ❀ ❀ ❀ ❀ ❀ = ?(✻) ?	.....

17	10, 11, 12, 1 + 0, 1 + 1, 1 + 2 1, 2, 3 1 + 2 + 3 = 6 ? = 6	10, 13, 14 …….. ……. …….. ? =	…..
18	$\underbrace{□□□}_{3}\ \underbrace{●●}_{2}\ \underbrace{ΔΔΔΔ}_{4}$ □ + ● − Δ 3 + 2 – 4 = ? ? = 1	$\underbrace{□□□}_{…}\ \underbrace{●●}_{…}\ \underbrace{ΔΔΔΔ}_{…}$ □ − ● + Δ … − … + … ? =	…..
19	8, 6, 2, 9, 1 → 1, 2, 6, 8, 9 9 – 1 = 8 7, 2, 4, 5, 6 → 2, 4, 5, 6, 7 7 – 2 = 5	8, 0, 9, 10, 1 → …,…, …,…,… … – … = ? ? =	…..
20			…..

WINTER – AGE GROUP 1

	 **EXAMINE**	 **THINK**	 **WRITE**
1	 → 3	 → ?	…..
2	✻ ✻ = ❀ ❀ ❀ ❀ ❀ ❀ ❀ ❀ ❀ = 1(✻) ? = 1	✻ ✻ ✻ = ❀ ❀ ❀ ❀ ❀ ❀ ❀ ❀ ❀ ❀ = ?(✻) ? = …	…..
3	 4 + 1 – 2 = ? ? = 3	 … + … – … = ? ? = …	…..
4	$1 \xrightarrow{+1} 2 \xrightarrow{+2} 4 \xrightarrow{+3} 7 \xrightarrow{+4} ? \xrightarrow{+5} 16$? = 11	$0 \xrightarrow{+1} \ldots \xrightarrow{+2} \ldots \xrightarrow{+3} \ldots \xrightarrow{+4} \ldots \xrightarrow{+5} ?$? = …	…..
5	$6 \xrightarrow{-1} 5 \xrightarrow{+2} 7 \xrightarrow{-3} 4 \xrightarrow{+4} ? \xrightarrow{-5} 3$? = 8	3, 2, 4, 1, 5, ? ? = …	…..
6	2, 3, 7 → 2 + 3 + 7 = 12 → 1 + 2 =? ? = 3	4, 5, 5 → ⋯ + ⋯ + ⋯ = ⋯ →… +… = → ? ? = …	…..
7	 1 + 4 + 5 = 10	 … + … + … = ? ? =…	…..
8	10, 11, 12, 13, 14 $8, 9, 10, \underbrace{\ldots}_{6}, 14$? = 6	10, 11, 12, 13, 14, 15, 16 $10, \underbrace{\ldots}_{?}, 16$? = …	……

9		? = …	……
10	1 + 1 + 1 = ? − 1 ? = 4	2 + 2 + 2 = ? − 2 ? = …	……
11	 □ = 3 (●) 2(★) = 1 (●)	2 (□) + 4 (★) = ? (●)	……
12	9 + 2 + 3 = 1? 9 + 2 + 3 = 14 → ? = 4	3 + 5 + 2 + 3 = 1? ? = …	……
13	 → 3  → 5  → 4	 ? = …	……
14	  1 + 2 + 3 + 4 + 5 = ? ? = 15	 … + … + … + … + … = ? ? = …	……
15	 ? → >	 ? → …	……

16	$\square = 5 - 2$ $\bigcirc = 8 - 4$ $\Delta = 2 + 3$ $3 < ? < 5$ $? \rightarrow \bigcirc$	$\square = 4 + 0$ $\bigcirc = 6 - 3$ $\Delta = 4 + 1$ $3 < ? < 5$ $? \rightarrow \ldots$	…..
17	$2,1,4 \rightarrow 1 < 2 < 4$ $1 + 4 - 2 = 3$ $? = 3$	$5,1,2 \rightarrow \ldots < \ldots < \ldots$ $\ldots + \ldots - \ldots = ?$ $? = \ldots$	…..
18	$8 + \bigstar < 10$ $\bigstar \rightarrow \underbrace{0,1}_{2} \rightarrow ? = 2$	$7 + \bigstar < 11$ $\bigstar \rightarrow \underbrace{\ldots\ldots}_{?} \rightarrow ? = \ldots$	…..
19	 ? $? = 3$	 ? $? = \ldots$	…..
20	$\ast < \ast < \odot$ $\ast + \ast + \odot = 5$ $\underbrace{0+1+4=5, 0+2+3=5}_{2}$ $? = 2$	$\ast < \ast < \odot$ $\ast + \ast + \odot = 6$ $? = \ldots$	…..

SPRING – AGE GROUP 1

Problem 1.

$$2 - 0 + 2 - 2 = \square$$

$$\square = ?$$

Problem 2.

$$10 - 2 - \square = 2$$

$$\square = ?$$

Problem 3.

$$1 + 1 + 2 + 2 - 3 - 3 = ?$$

Problem 4.

$$\square = 2$$

$$\Delta = \square + \square - 1$$

$$\square + \Delta = ?$$

Problem 5.

9, 12, 10 $\Longrightarrow$ 9 < 10 < 12$\Longrightarrow$ 91012 $\Longrightarrow$ 0

9, 7, 0, 13 $\Longrightarrow$ 0 < 7 < 9 <13$\Longrightarrow$ 07913 $\Longrightarrow$ 9

30, 9, 20, 10 $\Longrightarrow$ …. < … < …<…. $\Longrightarrow$ …. $\Longrightarrow$?

Problem 6.

9, 12, 10 $\Longrightarrow$ 9 < 10 < 12 $\Longrightarrow$ 10

9, 7, 0, 13, 1 $\Longrightarrow$ 0 < 1 < 7 < 9 <13 $\Longrightarrow$ 7

30, 9, 20, 19, 10 $\Longrightarrow$ … < … < … < … <… $\Longrightarrow$?

Problem 7.

5

+

7 – ■ = ●

=

9

● + ● – ■ = ?

Problem 8. How many different digits should we replace ☐ with, so that the following would be true? $100 - 10 - 20 < \square 0$

Problem 9.

$$10 + 20 + 30 + \square = 40 + 20 + 10$$

$$\square = ?$$

Problem 10.

$$\circ + \bullet + \square = 12;$$

$$\circ + \bullet = 7$$

$$\bullet + \square = 8$$

$$\bullet = ?$$

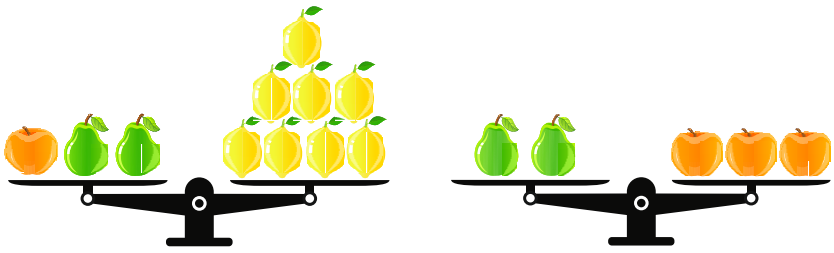
Problem 11. The sum of the one-digit number ✶ and 9 is equal to the sum of the one-digit number ✹ and 1. Calculate ✹ + ✶. Write down all possible sums.

$$✶ + 9 = ✹ + 1$$

$$✹ + ✶ = ?$$

Problem 12. Five children wrote down 10 + 20 in their notebooks. Then each child erased one digit and correctly calculated the sum.
How many of the resulting sums are definitely equal?

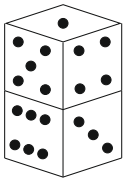
Problem 13. The total weight of one apple and two pears is equal to the weight of eight lemons. Two pears weigh as much as three apples. How many lemons are equal in weight to one apple?



Problem 14. I added two different two-digit numbers and I got a sum smaller than 23. Then I subtracted the smaller number from the greater number. What number did I get? Write down all possible answers.

Problem 15. April Fool's Day is celebrated on the 1st of April in many countries around the world. This year, April 1st falls on a Friday. How many Fridays are there in April 2022 after April Fool's Day?

Problem 16. How many dots are not visible?



Problem 17. There are apples and lemons in a fruit bowl. There are 6 apples, 2 of which are yellow. There are 11 pieces of yellow fruit in total. What is the total number of fruit in the bowl?

Problem 18. Which digit should we erase in 10 + 20 + 30, in order to get the smallest possible sum?

Problem 19. How many one-digit numbers are not greater than 7?

Problem 20. Calculate 1 – 10 + 2 + 3 + 4 + 5.

Example: 1 - 8 + 9 = 1 + 9 - 8 = 10 – 8 = 2

1 - 5 + 2 + 3 = 1 + 2 + 3 – 5 = 6 – 5 = 1

FINAL – AGE GROUP 1

Problem 1.

$$7 + 8 + 9 + 10 + \square = 8 + 9 + 10 + 11 \Longrightarrow \square = \cdots$$

Problem 2. Calculate

$$9 + 8 + 7 + 6 - 5 - 5 - 5 - 5 - 1$$

Problem 3. Calculate $\bigcirc + \square$ if

$$\bigcirc \underset{+2}{\rightarrow} 5 \underset{+\square}{\rightarrow} 11$$

Problem 4. How many numbers are missing? $0, 1, 2, 3, \underbrace{\ldots}_{?}, 19, 20$

Problem 5. I wrote down all numbers from 19 to 33. How many of these numbers have the digit 2?

Problem 6. The sum of two different one-digit numbers is 17. What will be the difference if we subtract the smaller number from the greater number?

Problem 7. How many integers are there that are NOT greater than 3?

Problem 8. If we decrease the minuend in the expression 11– 2 by 2, what difference will we get?

Problem 9.

$$10 + 23 = 24 \Longrightarrow 1 + 23 = 24 \Longrightarrow 0$$

$$14 + 23 = 24 \Longrightarrow \ldots. \Longrightarrow ?$$

Which digit should we remove from 14 + 23 = 24 in order for the equation to be correct?

Problem 10. What is the digit we must put in place of each of ☐, to get a true equation?

$$☐8 - 1☐ - 2 = 2☐$$

Problem 11. Place the numbers 1, 2 and 3 in the squares in such a way that the following would be true. ☐ > ☐ + ☐ – 2

Find the sum of the numbers in the grey squares ☐.

Problem 12. Maria drew three line segments. The first segment has a length of 11cm, the second segment is 4cm shorter than the first, and the third segment is 5cm longer than the second. How many centimeters is the length of the third segment?

Problem 13. Alex, Boris and Claire wrote down all numbers from 1 to 20 by taking turns: Alex wrote 1, Boris wrote 2, Claire wrote 3, Alex wrote 4, Boris wrote 5, Claire wrote 6, and so on. How many two-digit numbers did Claire write?

Problem 14. Alyssa has a thread with a length of 80 cm. She wants to cut off smaller pieces with lengths of 10 cm and 20 cm from the original thread. At most how many threads of both lengths can she get?

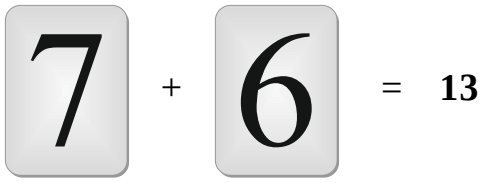
Problem 15. 1, 2, 3, 4, 5 and 6 dots are shown on the six faces of a die. By how many are the visible dots more than the hidden dots?



Problem 16. There are three cards with numbers written on them.

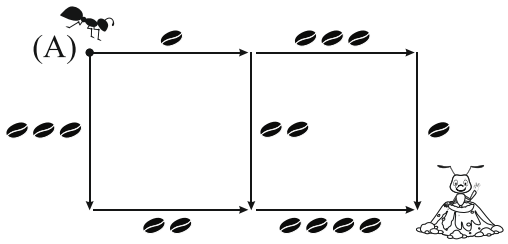


We can add two of them to get the number 13:



How many different results can we get if we calculate the sums of the numbers on two of these cards?

Problem 17. Freddie the Ant is moving from point A to the anthill by following the arrows (→) and collecting all the grains on the way. How many grains can she collect before she reaches the anthill? Write down all possible answers.



Problem 18. If I had another 9 sweets, I would have 11 sweets in total. How many more sweets do I need in order to have 20 sweets?

Problem 19. How many two-digit numbers are not greater than 19?

Problem 20. Calculate 12 – 13 + 8 + 2 + 7 + 3.

MATHEMATICAL RELAY – AGE GROUP 1

*The answer for each problem is represented by the symbols @, #, &, §, **
and must be used to solve each following problem. Each team must fill one answer sheet.

Problem 1. All numbers from 1 to 31, including 31, have been written in order. The digit 2 has been used @ times. Find @.

Problem 2. The sum of two one-digit numbers is @. The smallest possible value of the greater of the two numbers is #. Find #.

Problem 3. The numbers from 1 to 20 (including 20) have been written on a board. Igor erased the numbers which are not smaller than # and replaced each of them with the number 1. Then he calculated their sum and got & as a result. Find &.

Problem 4. A rope of length & cm is cut into § pieces of length either 11 cm or 13 cm. Find §.

Problem 5. Instead of adding a number to § and getting ***** as a result, Alex subtracted § from that number and got 17. Find *****.

ANSWER KEY – AGE GROUP 1

Problem	**1**	**2**	**3**	**4**	**5**
Answer	**13**	**7**	**35**	**3**	**23**

ANSWER KEY – AGE GROUP 1

Round / Problem	AUTUMN	WINTER	SPRING	FINAL
1	4	5	2	4
2	9	2	6	9
3	9	7	0	9
4	9	15	5	15
5	5	0	2	11
6	8	5	19	1
7	8	15	2	4
8	5	10	2	7
9	1	+ 2	10	4
10	9	8	3	3
11	13	8	8 and 10	3
12	6	3	2	12
13	1	2	2	3
14	9	16	1, 2	7
15	6	>	4	1
16	9	□	23	5
17	10	4	15	5, 7, 9
18	5	4	3	18
19	10	4	8	10
20	3	3	5	19

MATHEMATICS WITHOUT BORDERS
AGE GROUP 2

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5. The use of calculators or other electronic devices, as well as books containing formulae is NOT allowed during the course of the contest.
6. The problems contain natural numbers, as well as 0. (0, 1, 2, 3, 4, ..., 10, 11, ..., 99, 100, 101, ...)
7. For the problems containing a numerical answer, natural numbers, as well as 0 must be used. (0, 1, 2, 3, 4, ..., 10, 11, ..., 99, 100, 101, ...)
8. Students are NOT allowed to receive help by the Exams Officer or by anyone else during the contest.

WE WISH YOU ALL SUCCESS!

AUTUMN – AGE GROUP 2

Problem 1. What digit should we replace □ with, so that

$$50 = \square \text{ tens} + 20 \text{ ones?}$$

Problem 2. What number should we replace □ with, so that

$$31 + 29 = 95 - \square\,?$$

Problem 3. By how much is the sum of 31 + 13 greater than the difference of 31 – 13?

Problem 4. Find the missing number „?“.

$$100 - 40 = ? + 20.$$

Problem 5. What digit should we replace □ with, so that

$$7 \text{ tens} + 8 \text{ tens} + 50 \text{ ones} = \square \text{ hundreds?}$$

Problem 6. Find the missing number.

$$0, 5, 5, 10, 15, 25, \ldots, 65$$

Problem 7. How many two-digit numbers are smaller than 33?

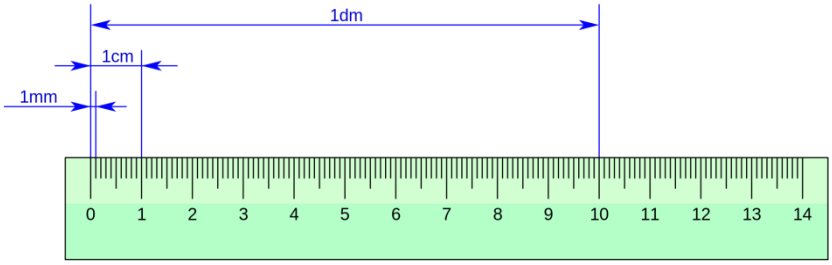
Problem 8. By how much is the sum of 20 + 40 + 80 greater than the sum of 30 + 40 + 50 ?

Problem 9. Which numbers are smaller than 15 and greater than 12?

Problem 10. Initially, George had 20 flowers. Nine of them were red, and the rest were yellow. He gave 7 yellow flowers to his mother. Then he gave 3 yellow flowers to his friend Maria. How many yellow flowers does he have left?

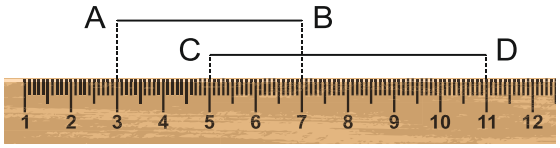
Problem 11. Harry planted 24 trees in a row. Each 2 trees were planted 1 meter apart. What is the length (in meters) of the row of trees that Harry planted?

Problem 12. A triangle has side lengths of 7 *cm*, 7 *cm* and 6 *cm*. A square has a side length of 1 dm. By how many centimetres is the perimeter of the square greater than the perimeter of the triangle?

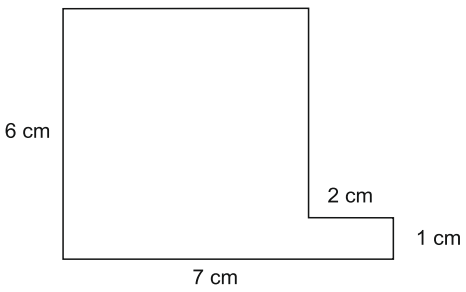


Problem 13. I have two sticks – one with a length of 15 cm and one with a length of 5 cm. I used them to measure the length of a board. I used the stick with a length of 15 cm three times, and I used the stick with a length of 5 cm once. Find the length of the board in cm.

Problem 14. Find the sum of the lengths of the line segments AB and CD in cm.



Problem 15. A rectangle was cut out of a larger rectangle, which resulted in a new figure. How many of the line segments of this figure have the same lengths?



Problem 16. Calculate

(7 + 9 + 11 + 13 + 15 + 17) – (17 + 15 + 13 + 9 + 7).

Problem 17. The first day of December 2021 falls on a Wednesday. Which day of the week will the 16^{th} day of December fall on?

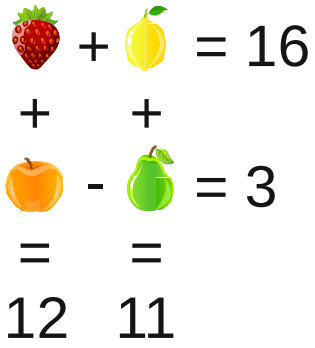
Problem 18. All the 33 lanterns were lit up in the castle. 11 of them were put out. How many lanterns are left?

Problem 19. We are given the smallest three-digit number made up of three different digits which has 1 as its tens digit. By how much is it greater than the smallest three-digit number made up of three different digits?

Problem 20. Calculate



if



WINTER – AGE GROUP 2

Problem 1. Calculate

90 – 80 + 80 – 70 + 70 – 60.

Problem 2. Calculate

2 + 8 + 12 + 18 – 20 – 20.

Problem 3. Calculate (15 – 5 – 4 – 3 – 2 – 1) + (1 + 2 + 3 + 4 + 5).

Problem 4. The minuend is equal to the difference. What is the subtrahend?

Problem 5. Calculate 40 – 1 + 20 – 2 + 30 – 3 + 20 – 4. How many tens are there in the result?

Problem 6. By how much is the sum of 19 + 20 + 21 + 22 + 8 smaller than the sum of 20 + 21 + 22 + 23 + 6?

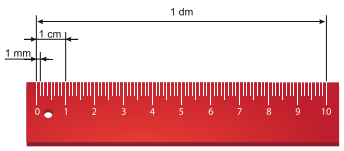
Problem 7. How many two-digit numbers are there in total that can be written in the square □ so that □ + 10 < 30 would be correct?

Problem 8. The minuend in the difference of 40 – 10 has been decreased by 10 and the subtrahend has been increased by 10. What is the new difference?

Problem 9. What digits should we replace ❄ with, so that the number 22 would not be smaller than the two-digit number ❄2?

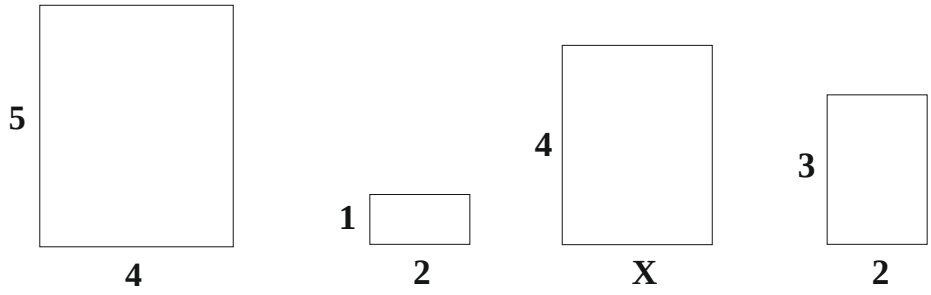
Problem 10. Find x, if the numbers 4, 8, 12, 16, 20, 24, 28,...., x have been written using 18 digits.

Problem 11. We have a tape with a length of 1 dm. By how many *millimetres* is it longer than a tape with a length of 9 cm?

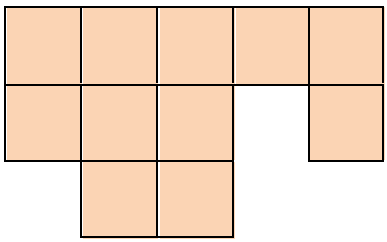


Problem 12. One of the sides of a rectangle has a length of 15 mm, and the other side is 20 mm longer. Find the perimeter of the rectangle in dm.

Problem 13. I divided a rectangle with sides 4 cm, 4 cm, 5 cm, 5 cm into three rectangles: the first with sides 1 cm, 1 cm, 2 cm and 2 cm; the second with sides 2 cm, 2 cm, 3 cm, 3 cm; the third with sides X cm, X cm, 4 cm, 4 cm. Find X.



Problem 14. The figure below is made up of 11 square tiles. What is the total number of squares in the figure?



Problem 15. There are 3 points on a straight line. Adam measured the distances between each two of the points and recorded two of them: 1 cm and 5 cm. Find the length of the third distance. Write down all possible answers.

Problem 16. How many points should we place on a straight line in order to get exactly 6 line segments?

Problem 17. We can use three different weights of 1 kg, 2 kg and x kg in order to weigh each of ten packages weighing between 1 kg and 10 kg on a pair of scales. Find the number x.



Problem 18. Ivo and Ellie wrote down eleven numbers each.
Ivo wrote down 2, 5, 8, ..., 29, 32. Ellie wrote down 11, 14, 17, ..., 38, 41.
How many of the numbers written down by Ivo were also written down by Ellie?

Problem 19. Several children have a total of 20 balloons. Five of the children have three balloons each, and each of the remaining children has one balloon. How many children are there?

Problem 20. What is the greatest difference of two two-digit numbers made up of 4 different digits?

SPRING – AGE GROUP 2

Problem 1.

$$10 \text{ tens} - 10 \text{ ones} = \square$$
$$\square = ?$$

Problem 2.

$$96 - (31 - 15) = \square - (43 - 27)$$
$$\square = ?$$

Problem 3.

$$38 \text{ minutes} + 47 \text{ minutes} - \square \text{ minutes} = 1 \text{ hour}$$
$$\square = ?$$

Problem 4. Calculate

$$(4 + 5 + 6 + 21 + 32 + 43) - (14 + 25 + 36 + 1 + 2 + 3)$$

Problem 5. Find ■, if

$$9 + 9 + 9 + 9 + 4 = 4 \times (4 + ■).$$

Problem 6. The minuend in the difference of 60 – 10 has been decreased by 19 and the subtrahend has been increased by 11. What is the new difference?

Problem 7. How many numbers are missing?

$$7, 14, 21, \ldots, 84, 91, 98$$

Problem 8. The sum of two numbers is 29. One of the numbers is greater than the other by 11. What is the greater number?

Problem 9. How many of the symbols „+“ must be replaced with the multiplication symbol „×“, so that the equation would be correct?

$$2 + 3 + 5 + 6 = 6 + 6$$

Problem 10. Calculate

$$1 - 10 \times 10 + 2 \times 5 + 4 \times 5 + 6 \times 5 + 8 \times 5.$$

Example: 1 - 8 + 9 = 1 + 9 - 8 = 10 – 8 = 2

1 - 5 + 2 + 3 = 1 + 2 + 3 – 5 = 6 – 5 = 1

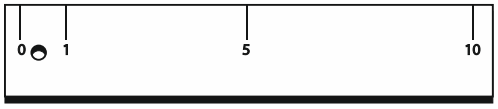
Problem 11. I wrote down 21 numbers. I erased a few of them and wrote down their sum. The numbers are now 16. How many numbers have been erased?

Problem 12. How many of the numbers among 0, 1, 2, …, 89, 90, 91 can we present as the product of two consecutive numbers? ***Example:*** $20 = 4 \times 5$; $56 = 7 \times 8$.

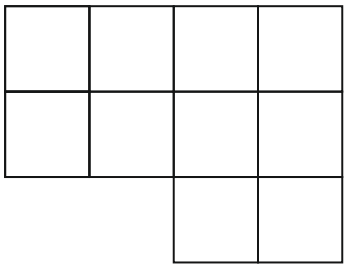
Problem 13. It takes 6 seconds for a robot to cut a pipe into 2 parts. How many seconds will it take for the robot to cut the same pipe into 8 parts?

Problem 14. A square with a perimeter of 20 cm has been cut into two rectangles. Find the sum of the perimeters of these two rectangles in centimetres.

Problem 15. Look at the ruler below. How many different lengths can be measured by this ruler in one setting?



Problem 16. How many squares are there in the picture?



Problem 17. Three different points A, B and C lie on a straight line. There are 3 line segments with two of the points A, B and C as their endpoints $(AB, BC$ and $AC)$.

If $AB + BC + AC = 32\ cm$, find the length of the longest of the three line segments.



Problem 18. In how many ways can we express the number 32 as the product of different multipliers without considering their order?

Problem 19. There is a total of 54 roses in 10 bouquets of roses. Some of the bouquets have 5 roses, and the others have 7 each. How many of the bouquets have 5 roses each?

Problem 20. You have 5 cards with the numbers 0, 3, 6, 7, 9 written on them. How many different numbers greater than 680 and smaller than 720 can be created using these cards?



FINAL – AGE GROUP 2

Problem 1. Calculate

$$(1 + 1 + 1 + 1 + 1 + 1 + 1 + 1) \div (1 + 1 + 1 + 1) - 1 .$$

Problem 2. How many of the numbers 1, 3, 5 and 7 can be written in the square □ so that the following would be correct?

$$\square + 19 > 5 + 18$$

Problem 3. Find □, if $6 + 6 + 6 + 9 = \square \times 9$.

Problem 4. Place the same number in all the squares □ so that the following would be correct:

$$\square + 5 + 2 \times 5 = \square \times 4.$$

What is the number?

Problem 5. Which digit should we remove in order for the equation to be correct?

$$10 \times 25 + 10 \times 2 = 40$$

Problem 6. Insert brackets to make the equation correct.

$$3 \times 4 - 2 + 4 = 2 \times 9.$$

Write down the whole expression in the answer sheet.

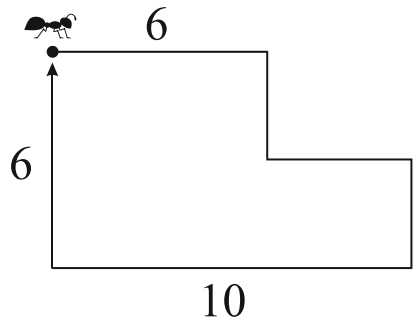
Problem 7. A pink ice cream costs 2 euro, and a white ice cream costs 3 euro. How many euro would it cost me to buy 7 pink ice creams and 5 white ice creams for my friends?

Problem 8. How many minutes should be subtracted from 150 seconds in order to get 30 seconds?

Problem 9. In 9 years Claire will be 4 times her current age. What will be Claire's age 6 years from now?

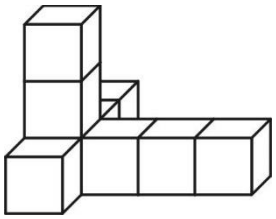
Problem 10. How many even two-digit numbers are divisible by 7?

Problem 11. An ant was walking along the sides of the figure below. Its walk ended right where it started. The distance that the ant travelled is equal to the perimeter of a square with a side length of x cm. Find x.

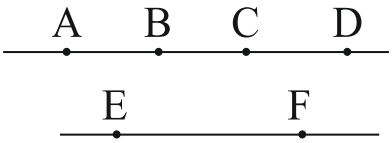


Problem 12. The square A and the rectangle B can be combined to form the rectangle C, which has a perimeter of 40 cm. If the perimeter of the square A is 28 cm, find the smaller side of the rectangle B in cm.

Problem 13. The figure below is made up of 9 cubes equal in weight and weighs 24 grams. What is the weight of a figure made up of 6 cubes?



Problem 14. The points A, B, C and D lie on a straight line. How many triangles have three of the six points A, B, C, D, E and F as their vertices?



Problem 15. I cut out 12 figures – triangles and squares. The number of triangles is three times the number of squares. How many vertices in total do all the figures have?

Problem 16. Claire wrote down all one-digit odd numbers from the smallest to the greatest. Then, in between each pair of neighbouring numbers, she wrote down the product of those numbers. Find the sum of all numbers.

Problem 17. On Monday, there were 2 water lilies in a lake. Each following day of the week, the number of lilies would double compared to the previous day. How many lilies were there in the lake at the end of the week?

Problem 18. The following information is known about three numbers. Their sum is 20. The second number is equal to half of the first number. The first number is twice the third number. Find the first number.

Problem 19. Peter wrote the numbers 1, 2, 3, 4 and 5 in his notebook as follows:

- he wrote 1 next to 2, to the right of 2;
- he wrote 5 next to 3, to the right of 3;
- he wrote 4 next to 1, to the right of 1;
- he wrote 3 next to 4, to the right of 4.

After that he placed the following symbols in between the numbers, from left to right: "÷", "×", "–" and "+". He then calculated them correctly. What result did he get?

Problem 20.. The sum of 7 consecutive numbers is a number divisible by 2. How many of these numbers are odd?

MATHEMATICAL RELAY - – AGE GROUP 2

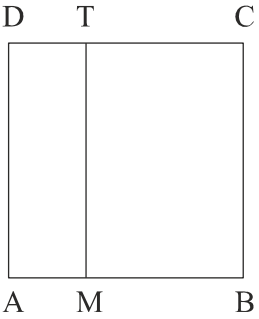
The answer for each problem is represented by the symbols **@, #, &, §** and *****, *and must be used to solve each following problem. Each team must fill one answer sheet.*

Problem 1. 4 different digits have been used to write down two odd two-digit numbers with a difference of 2. The greatest possible sum of the two numbers is **@.** Find **@**.

Problem 2. It takes the ant **@** seconds to travel a third of the way. It can travel the whole way in # minutes. Find #.

Problem 3. The product of five numbers is #, and their sum is the odd number **&**. Find **&**.

Problem 4. The side of the square *ABCD* is **&** cm. The rectangle *AMTD* has a side *AM*, which is a third of the side *AD*. The square *ABCD* has a perimeter, which is greater than the perimeter of the rectangle *MBCT* by **§** cm. Find **§**.



Problem 5. I chose 4 numbers from 1 to §, such that the product of two of the chosen numbers would be equal to the product of the other two chosen numbers. The sum of the numbers that have not been chosen is *. Find all possible values of *.

MATHEMATICAL RELAY – AGE GROUP 2

ANSWER KEY

Problem	**1**	**2**	**3**	**4**	**5**
Answer	**160**	**8**	**9**	**6**	**6; 9**

ANSWER KEY - AGE GROUP 2

Round / Problem	AUTUMN	WINTER	SPRING	FINAL
1	3	30	90	1
2	35	0	96	2
3	26	15	25	3
4	40	0	30	5
5	2	10	6	5
6	40	2	20	$3 \times (4 - 2 + 4)$
7	23	10	8	29
8	20	10	20	2
9	13, 14	1 or 2	3	9
10	1	40	1	7
11	23	10	6	8
12	20	1	10	6
13	50	3	42	16
14	10	14	30	16
15	2	4 or 6	5	39
16	11	4	14	141
17	Thursday	7	16	128
18	33	8	5	10
19	108	10	8	10
20	5	88	6	4

MATHEMATICS WITHOUT BORDERS

AGE GROUP 3

INSTRUCTIONS

1. Please **DO NOT OPEN** the contest papers until the Exams Officer has given permission.
2. There are 20 open-ended questions in the test. Please write down your answers in the **ANSWER SHEET.**
3. Each problem is scored 2 points for a correct answer; 1 point if there are two or more answers and at least half are given, or if one wrong answer is given in addition to the correct answer; 0 points for a wrong answer or no answer.
4. Working time: not more than 60 minutes. In the case of an equal number of points, the higher ranked participant will be the one who has spent less time solving the problems.
5. The use of calculators or other electronic devices, as well as books containing formulae is NOT allowed during the course of the contest.
6. The problems contain natural numbers, as well as 0. (0, 1, 2, 3, 4, ..., 10, 11, ..., 99, 100, 101, ...)
7. For the problems containing a numerical answer, natural numbers, as well as 0 must be used. (0, 1, 2, 3, 4, ..., 10, 11, ..., 99, 100, 101, ...)
8. Students are NOT allowed to receive help by the Exams Officer or by anyone else during the contest.

WE WISH YOU ALL SUCCESS!

AUTUMN – AGE GROUP 3

Problem 1. Calculate $36 - 18 \div 3 + (36 - 18) \div 3$.

Problem 2. Calculate $3 \div 1 + 6 \div 2 + 9 \div 3 + 12 \div 4 + 15 \div 5 + 18 \div 6 + 21 \div 7$.

Problem 3. Which two different symbols among +, - , × and ÷ should we place instead of □, in order for the following equation to be correct?

$$12 \square 3 \square 1 = 4$$

Problem 4. Calculate

$$2 \times 10 - 19 + 2 \times 9 - 17 + 2 \times 8 - 15 + 2 \times 7 - 13 + \ldots + 2 \times 2 - 3 + 2 \times 1 - 1$$

Problem 5. John placed brackets in the following equation to make it correct. Find the value of the expression in the brackets.

$$12 - 9 + 5 \div 4 = 2$$

Problem 6. How many digits are needed to write down the even two-digit numbers smaller than 29?

Hint: We use two digits to write the number 22.

Problem 7. What digit should we replace □ with, so that the following would be true?

$$2 \times \square + 3 \times \square > 40$$

Problem 8. What is the sum of the digits of the greatest number of the fourth hundred?

Problem 9. Which of the symbols "<", ">" and "=" should we replace ● with, so that the following would be correct?

10 hundreds - 12 ones ● 9 hundreds + 8 tens + 18 ones

Problem 10. Let us choose two of the numbers 1, 2, 3 and 6 and multiply them. How many different results can we get?

Problem 11. I wrote down two numbers: 2 and 4. After those two numbers, I wrote down another three numbers: a, b, c. If $2 \times 4 \times a = 4 \times a \times b = a \times b \times c$ and $a + b + c < 8$, find the number a.

Problem 12. The perimeter of a square is 6 cm. Find its side length in millimetres.

Problem 13. Ivan is 174 cm tall. Todor is 15 cm shorter than Ivan, and Ivan is 6 cm taller than Dobri. By how many cantimetres is Dobri taller than Todor?

Problem 14. I have a rectangular sheet of paper with a thickness of 1 mm. I started folding the sheet until the thickness became 16 mm. How many times did I fold it?

Problem 15. What number should we replace x with, so that the following would be correct?

$$\big((x + 2) \div 2\big) \div 3 = 3$$

Problem 16. Find the difference of the digits □ and Δ, if □Δ - Δ□ = 18.

□ - Δ = ?

Problem 17. How many feet do three sparrows and four rabbits have in total?



Problem 18. Find the sum of the first 8 numbers of the following sequence.

0, 1, 10, 11, 100, 101, …, …, 1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111

Problem 19. There are as many even numbers from 1 to 9 as there are odd numbers from 10 to the even number X. Find the number X.

Problem 20. The product of 3 different natural numbers is 12. What is the least possible value of the greatest of these numbers?

WINTER – AGE GROUP 3

Problem 1. Calculate $99 - 99 \div 3 + (99 - 96) \div 3$.

Problem 2. Calculate $100 \div (10 - 9 + 9 - 8 + 8 - 7 + 7 - 6 + 6 - 5)$.

Problem 3. Calculate the ones digit of the number equal to $2 \times 3 \times 9 - 9$.

Problem 4. If $(36 \div 9) \div \circ = 36 \div (9 \div \circ)$, what number should we replace $\circ$ with?

Problem 5. How many digits 1 are necessary to write down the numbers from 99 to 134?

$$\underbrace{99, 100, \ldots, 133, 134}_{36 \text{ numbers}}$$

Problem 6. What is the smallest two-digit number which is a factor of the number equal to

$$1 \times 13 + 4 \times 13 + 7 \times 13?$$

Problem 7. How many of the numbers from 1 to 111 are divisible both by 2 and by 5?

Problem 8. The product of two 2-digit numbers $\overline{ab}$ and $\overline{cd}$ is equal to 7×39.

$$\overline{ab} \times \overline{cd} = 7 \times 39$$

Calculate $\overline{ab} + \overline{cd}$.

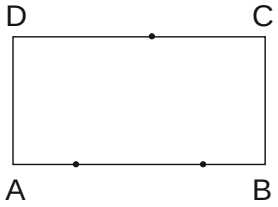
Problem 9. The sum of 3 different odd numbers is 21. What is the greatest possible value of the smallest among the numbers?

Problem 10. The product of 10 natural numbers is 10. What are the possible sums of these numbers?

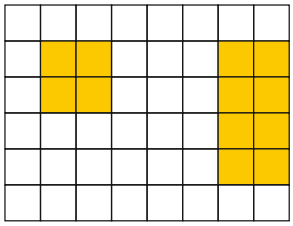
Problem 11. One of nine coins weighing 147 grams in total is 3 grams heavier than each of the other eight. How many grams does the heaviest coin weigh?

Problem 12. A rectangle is made up of 3 squares, each with a perimeter of 12 cm. How many centimeters is the perimeter of the rectangle?

Problem 13. The points *X* and *Y* are on the side *AB* of the rectangle *ABCD*, and the point *Z* is on the side *CD*. How many parts is the rectangle divided into by the segments *DX*, *CY* and *AZ*? Give all possible answers.



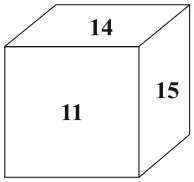
Problem 14. Two shaded rectangles are drawn on a square grid, one of which is a square. The perimeter of the square is 48 cm. Find the perimeter of the other shaded rectangle.



Problem 15. A box contains 18 colored pens: 3 black, 4 blue, 5 red and 6 green. Alex wants to take a few pens from the box without looking. At least how many pens should he take out (without looking), so that there would be at least 4 pens of the same color?

Problem 16. Let *A* and *B* be two different digits and let $A > B$. The product of the two-digit number *AB* and the one-digit number *B* is less than 20.
Which is the greatest possible sum of $A + B$?

Problem 17. Six consecutive natural numbers have been written on the six faces of a cube. The figure shows that the numbers 15, 11 and 14 have been written on three of the faces of this cube. Calculate all possible sums of the six numbers.



Problem 18. We have a thread with a length of 123 cm. If we cut a few pieces from it, each with a length of 11 cm, there will be 2 cm left of the thread. How many parts has the thread been cut into?

Problem 19. John had to multiply a number by 7 and then add 3 to the product. However, he got confused, added 7 to the number, and then multiplied the sum by 3. He got 123. Find the original answer John was supposed to get.

Problem 20. Calculate

$$(12 \times 2 - 12 \div 2 + 14 \times 2 - 14 \div 2 + 16 \times 2 - 16 \div 2) \div 9 - 7.$$

SPRING – AGE GROUP 3

Problem 1. Calculate $6 \times 124 - 124 \times 3 + 28$

Problem 2. What number is behind the square □?

□ × 10 – 10 – □ = 80

Problem 3. Calculate the following in minutes.

1 hour – 50 minutes – 120 seconds

Problem 4. Calculate 2 + 3 + 26 + 37 + 58 + 79 + 21 + 32 + 63 + 74 + 97 + 98

Problem 5. If we erase one digit in each of the two-digit numbers 28, 39 and 76, and multiply the resulting numbers, we will get the number A. What is the smallest possible value of A?

Problem 6. You are given a number. If you subtract the product of 12 and 5 from that number, you will get the product of 15 and 4. What is the number?

Problem 7. Find the number x if $31 \times x$ is a number from 63 to 100.

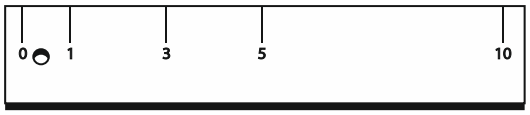
Problem 8. Find the ones digit of A if $A = 119 \times 9 + 225 \times 5 + 326 \times 6$

Problem 9. The natural number B is 7 times smaller than the two-digit number A and is greater than the natural even number C by 11. What is the number A?

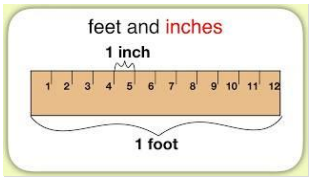
Problem 10. By how much is the product of five different odd one-digit numbers greater than the product of five different even one-digit numbers?

Problem 11. The perimeter of an equilateral triangle is 1 meter 2 decimeters and 3 centimeters, and the perimeter of a square is 1 meter and 8 centimeters. By how many centimetres is the side of the square smaller than the side of the triangle?

Problem 12. Look at the ruler below. How many different lengths can be measured by this ruler in one setting?



Problem 13. A rope with a length of 9 feet has been cut into 6 equal parts. Find the length of each part in inches. (1 foot is equal to 12 inches)



Problem 14. There are two numbers that express the sums of exactly three sides of a rectangle that is not a square. If they are equal to 17 cm or 19 cm, find the perimeter of this rectangle in cm.

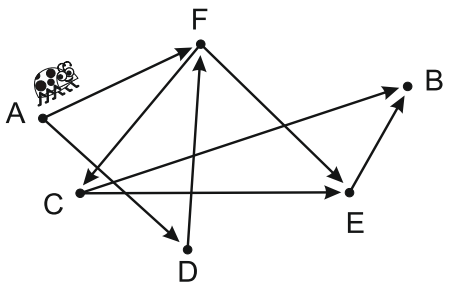
Problem 15. What is the sum of the natural numbers a, b and c, if the smallest among them is 4 and $4 \times a = 6 \times b = 15 \times c$?

Problem 16. The dividend is greater than the divisor by 48, the quotient is 7, and the remainder is 0. Find the dividend.

Problem 17. The two-digit numbers x and $x + 1$ have been written using four different digits and their product is a three-digit number. What is the number x? Write down all answers.

Problem 18. John wrote down all two-digit numbers with different even digits on some cards. At least how many cards does Peter have to choose, without looking, in order to be sure that he's picked a card with a number divisible by 5?

Problem 19. A ladybird is moving from point A to point B following the direction of the arrows. In how many ways can it get to point B?



Problem 20. The following three actions have been applied to the number 9 in a random order:

- multiplication by 3;
- division by 3;
- addition with 3.

How many possible results are there?

FINAL – AGE GROUP 3

Problem 1. Find □, if

$$999 - 99 \times 9 = \square \times 9$$

Problem 2. Place the same number in all squares □ so that the following would be correct.

$$\square + \square + \square + \square = \square \times \square$$

What is this number? Write down all possible answers.

Problem 3. Calculate

$$100 \div (21 - 17 + 17 - 13 + 13 - 9 + 9 - 5 + 5 - 1)$$

Problem 4. Calculate

$$5 \times 25 - 3 \times 4 - 4 \times 5 - 5 \times 6 + 2 \times 6 + 2 \times 10 + 2 \times 15$$

Problem 5. Find the ones digit of A, if:

$$A = 2 \times 5 + 4 \times 15 + 6 \times 25 + 8 \times 35$$

Problem 6. How many times is the number equal to $100 - 25 \div 5$ smaller than the number equal to $1000 - 250 \div 5$?

Problem 7. How many of the symbols "+" must be replaced with the symbol " × ", so that the equation below would be correct?

$$0 + 1 + 2 + 3 + 4 + 5 = 60$$

Problem 8. Replace the stars in the equation $26 \times * = **$ with different digits, so that it would be correct. Calculate the possible sums of the three numbers of the equation.

Problem 9. It takes 7 seconds for a robot to complete a product. How many such products can the robot complete in 107 seconds?

Problem 10. How many of the numbers from 1 to 199 have the smallest possible product of their digits?

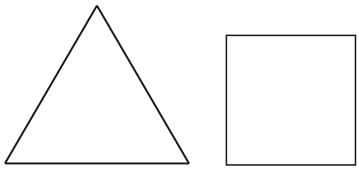
Problem 11. The sum of the dots on the opposite faces of a die is 7. Two identical dice have been stuck to each other along the faces that have the same number of dots. What could be the number of dots on the face marked with the letter A?



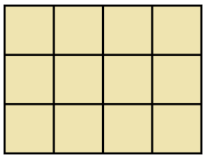
Problem 12. A point C is marked on a segment AB of length 52 cm such that AC = 16 cm. At least how many more points should we mark on the segment AB so that the distances between any two adjacent points on it, including the ends of the segment, are equal?

Problem 13. A square with a side length of 8 cm was divided into 64 smaller squares, each with a side length of 1 cm. Two of the rows and two of the columns were coloured in blue. The remaining part was coloured in red. By how many are the red squares more than the blue squares?

Problem 14. The perimeter of an equilateral triangle is 3 cm smaller than the perimeter of a square with a side length of 75 mm. Find the length of the side of the triangle in cm.



Problem 15. Twelve identical squares with a side length of 1 cm have been used to form a rectangle with a side length of 4 cm. How many of the rectangles are not squares?



Problem 16. One or more of the digits in the sum 12 + 34 + 56 + 78 were replaced with the digit 9, so that the new sum would be equal to 210. Find the sum of the replaced digits.

Problem 17. If $A = 3 \times 3 \times 3 \times 3 \times 3 \times 3$ and $B = 6 \times 6 \times 6 \times 6 \times 6 \times 6$, calculate $B \div A$.

Problem 18. Calculate □ - ☺ + ○, if

$$3 \xrightarrow{\times 8} \square \xrightarrow{\div 6} ☺ \xrightarrow{\times ○} 56.$$

Problem 19. There are 4 green and 3 yellow points on a plane. Line segments are drawn with each two points. How many of these line segments have endpoints of the same colour?

Problem 20. The product of the natural numbers from 1 to N is equal to the product

$$\underbrace{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}_{8} \times \underbrace{3 \times 3 \times 3 \times 3}_{4} \times \underbrace{5 \times 5}_{2} \times 7$$

Find N.

MATHEMATICAL RELAY - AGE GROUP 3

*The answer for each problem is represented by the symbols @, #, &, § and * and must be used to solve each following problem. Each team must fill one answer sheet.*

Problem 1. The sum of the dots on opposite faces of a standard die is 7. If we glue 3 dice on top of each other, the largest possible sum of the dots that are on the unglued faces is @. Find @.

Problem 2. There are # two-digit numbers that divide the number @ and leave a remainder of 0. Find #.

Problem 3. A square with a perimeter of 108 cm is formed of identical squares with side lengths of # cm. If we erase # rows and # columns, there will be & squares with a side length of # cm left. Find &.

Problem 4. I have both three-leaf and four-leaf clovers with a total of & petals. Their total number is equal to the even number §. Find §.

Problem 5. The number § is presented as the sum of * numbers. The greatest possible product of these numbers is 36. Find *.

MATHEMATICAL RELAY - AGE GROUP 3

ANSWER KEY

Problem	**1**	**2**	**3**	**4**	**5**
Answer	**54**	**3**	**36**	**10**	**3; 4**

ANSWER KEY - AGE GROUP 3

Round / Problem	AUTUMN	WINTER	SPRING	FINAL
1	36	67	400	12
2	21	20	10	0 or 4
3	÷ ; ×	5	8	5
4	10	1	590	125
5	8	49	36	0
6	20	12	120	10
7	9	11	3	4
8	21	34	2	53; 107
9	<	5	91	15
10	5	19, 15	945	28
11	0 or 1	19	14	2
12	15	24	8	11
13	9	5, 6	18	8
14	4	72	24	9
15	16	13	29	40
16	2	9	56	15
17	22	75, 81	19 and 29	64
18	444	12	13	34
19	18	241	6	9
20	4	0	3	10

MATHEMATICS WITHOUT BORDERS
AGE GROUP 4

INSTRUCTIONS

1. Please **DO NOT OPEN** the contest papers until the Exams Officer has given permission.
2. There are 20 open-ended questions in the test. Please write down your answers in the **ANSWER SHEET.**
3. Each problem is scored 2 points for a correct answer; 1 point if there are two or more answers and at least half are given, or if one wrong answer is given in addition to the correct answer; 0 points for a wrong answer or no answer.
4. Working time: not more than 60 minutes. In the case of an equal number of points, the higher ranked participant will be the one who has spent less time solving the problems.
5. The use of calculators or other electronic devices, as well as books containing formulae is NOT allowed during the course of the contest.
6. The problems contain natural numbers, as well as 0. (0, 1, 2, 3, 4, ..., 10, 11, ..., 99, 100, 101, ...)
7. For the problems containing a numerical answer, natural numbers, as well as 0 must be used. (0, 1, 2, 3, 4, ..., 10, 11, ..., 99, 100, 101, ...)
8. Students are NOT allowed to receive help by the Exams Officer or by anyone else during the contest.

WE WISH YOU ALL SUCCESS!

AUTUMN – AGE GROUP 4

Problem 1. Calculate $333 - (33 - 13) - 222 + 33 - 13$

Problem 2. Calculate $923 - (5 \times 6 - 6)$

Problem 3. Which number should we replace ☺ with, in order for the following equation to be correct?

$$☺ \times 25 = 1000 \div 8.$$

Problem 4. Place brackets in the following expression to make it correct:

$261 \div 2 + 21 + 6 = 9.$

What is the divisor?

Problem 5. If we remove a digit from the following product, we will get the smallest possible product. What digit should be removed?

$$2 \times 5 \times 8 \times 11 \times 14$$

Problem 6. Calculate $(6 \times 6 \times 6 \times 6 \times 6 \times 6) \div (2 \times 2 \times 2 \times 3 \times 3 \times 3)$.

Problem 7. How many zeros are in the number thirty two million five hundred and two thousand?

Problem 8. Find the tens digit of the number equal to

$$2 \times 2 \times 5 \times 5 \times 5 + 2 \times 2 \times 2 \times 2 \times 2 \times 5.$$

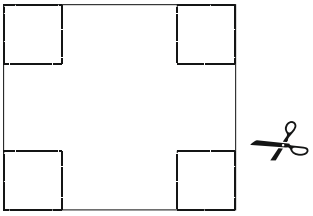
Problem 9. How many numbers are missing from the following expression?

$$11 + 13 + \ldots + 41 + 43$$

Problem 10. How many digits are there in the number that, when divided by 6, results in a quotient of 16 and an unknown remainder?

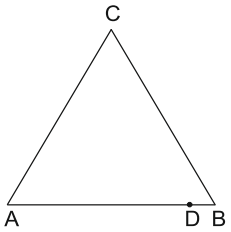
Problem 11. I used a piece of thread to form a square with a side length of 12 cm. Then I used the same piece of thread to form a triangle with equal sides. Find the side length of the triangle in cm.

Problem 12. A rectangle has a length of 8 cm and a width of 7 cm. A square has been cut out from each of the four corners of the rectangle. Each of the squares has a perimeter of 8 cm. How many square centimeters is the area of the resulting figure?

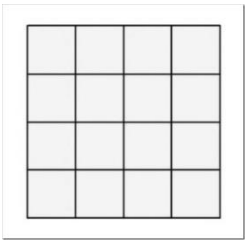


Problem 13. The points A, B, C and D have been placed along a straight line, such that $AB = 3$, $BC = 4$, $AD = 2$, $CD = x < 5$. Find x.

Problem 14. $\Delta\ ABC$ is an equilateral triangle with a perimeter of 24 cm. The point D is on the side AB, such that the length of the line segment AD is seven times greater than the length of the line segment BD. Find the length of the line segment BD in centimetres.



Problem 15. A square with a side length of 4 cm has been divided into 16 squares, each with a side length of 1 cm.



After cutting along the sides of some of these 16 squares, Ivo got a rectangle made up of 8 squares. Find the perimeter of this rectangle in centimetres.

Problem 16. The number 23 has been written on a board. Every 12 minutes this number is erased and replaced by the number equal to the product of its digits added to 23. For example, after 12 minutes, $2 \times 3 + 23 = 29$ will be written on the board. What number will be written on the board after 1 hour?

Problem 17. How many natural numbers x are there, such that the product of $19 \times x$ is an even three-digit number smaller than 400?

Problem 18. Both numbers 15 and 21 leave a reminder of 3 when divided by the natural number n. Find n.

Problem 19. What is the remainder left after dividing 222219 by 9?

Problem 20. What is the greatest number that is divisible by 40 and is made up of different digits?

WINTER – AGE GROUP 4

Problem 1. Calculate

20 222 021 – 20 210 000 – 2 022

Problem 2. Calculate

111 ÷ 3 + 222 ÷ 6 + 333 ÷ 9 + 444 ÷ 12

Problem 3. Calculate

2020 + 2021 + 2022 + 2023 – (1999 + 2000 + 2001 + 2002)

Problem 4. The difference of two three-digit numbers has been written on a piece of paper. Two of the digits have been replaced with □ and ■. Find the two-digit number ■□.

641 - 27□ = 3■4

Problem 5. Find the greatest 4-digit number that is made up of different digits and is smaller than 2901.

Problem 6. What is the digit behind ● if 3●7 × 6 = 2022?

Problem 7. Find x.

$$(2022 - 2021) - (2022 - x) = 1.$$

Problem 8. How many two-digit numbers are divisible by 3 but not by 6?

Problem 9. How many digits can we place instead of *, so that the inequality would be correct?

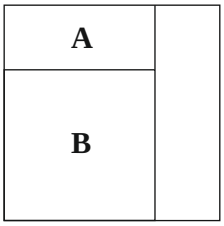
6*37 < 6373

Problem 10. Which symbol from the roman numerals should we remove from the expression XL + XIII – L in order to get II as a result?

Problem 11. How many rectangles in total are there in the figure that contain at least one ant?



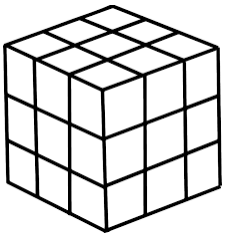
Problem 12. A small square *B* with a perimeter of 12 cm and a rectangle *A* have been cut from a large square with a perimeter of 20 cm. How many centimeters is the perimeter of rectangle *A*?



Problem 13. The side length of a square with a perimeter of 2 m is 10 cm greater than the side length of an equilateral triangle. Find the perimeter of the triangle in decimeters.

Problem 14. I placed two more points on a line segment and got 6 line segments in total. At least how many more points do I have to place on the line segment to get 15 line segments?

Problem 15. A large cube has been built from 27 identical small cubes, painted white. The faces of the large cube have been painted red. How many of the faces of the small cubes have been left white?



Problem 16. What is the greatest four-digit number which has 70 as the product of its digits?

Problem 17. How many two-digit numbers $\overline{ab}$ are there in total, if $a < b$?

Problem 18. Find all three-digit numbers which, when divided by 11, will have an equal quotient and remainder.

Problem 19. Ivo divided all natural numbers smaller than 16 into several groups so that the sum of the numbers in each group would be the same number x, which is smaller than 27. Find all possible values of x if there are at least two numbers in each group.

Problem 20. There is a total of 120 kg of apples in four crates. The total weight of the apples in the first two crates is 54 kg, and the total weight of the apples in the second, third and fourth crates is 96 kg. How many kilograms of apples are there in the second crate?

SPRING – AGE GROUP 4

Problem 1. Calculate

$$37 \times 120 + 74 \times 30 - 111 \times 60$$

Problem 2. Calculate

$$(225 \times 7) \div 5 + (222 \times 14) \div 37 - 56 \times 7.$$

Problem 3. Calculate $\mathbf{O} - 240$ if

$$(150\,000 - 30\,000) \div \mathbf{O} = 240.$$

Problem 4. What digit should we replace * with?

$$1 * 3 \times * 4 = 2952$$

Problem 5. Calculate $2020 \div 101 + 3030 \div 303 - 8080 \div 404$.

Problem 6. Remove the same digit in the three factors 128, 281, and 821 of the expression $128 \times 281 \times 821$, such that the greatest possible product would be obtained. Which digit was removed?

Problem 7. Find the ones digit of A if

$$A = 11 \times 12 + 11 \times 12 \times 13 + 11 \times 12 \times 13 \times 14 + 11 \times 12 \times 13 \times 14 \times 15$$

Problem 8. Calculate x if

$$\Big(\big((x \div 2) \times 3\big) \div 4\Big) - 23 = 4$$

Problem 9. Find the digit x.

$$\underbrace{99989796\ldots x}_{40\text{ digits}}$$

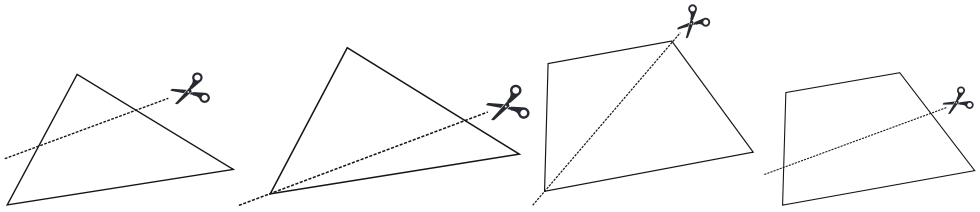
Problem 10. John chose a digit and wrote down the greatest two-digit number A and the smallest two-digit number B, each with different digits, where the chosen digit is the ones digit. Find all possible values of the difference of $A - B$.

Problem 11. Pippi must travel a distance of 2 km 100 m and 6 cm. She divided the length of her journey into 18 equal parts. Find the length of 1 part in *cm*.

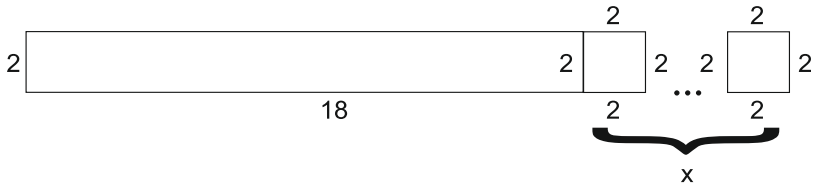
Problem 12. A rectangle has two sides with lengths a cm and b cm ($a \neq b$). The side a is $1/10^{\text{th}}$ the size of the perimeter of the rectangle. Calculate $b \div a$.

Problem 13. A triangle was cut into two shapes, along a straight line. Then, one of the shapes was cut in the same way into two shapes, and so on. At least how many cuts will it take for the number of vertices of the resulting figures to be 28?

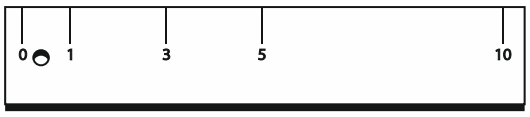
Examples of cutting in a straight line:



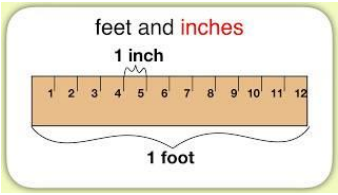
Problem 14. x squares with a side length of 2 cm have been placed to the right of rectangle A, which has side lengths of 2 cm and 18 cm. The resulting figure is the rectangle B with sides 2 cm and y cm, and a perimeter twice the perimeter of A. Calculate $x + y$.



Problem 15. Look at the ruler below. How many different lengths can be measured by this ruler in one setting?



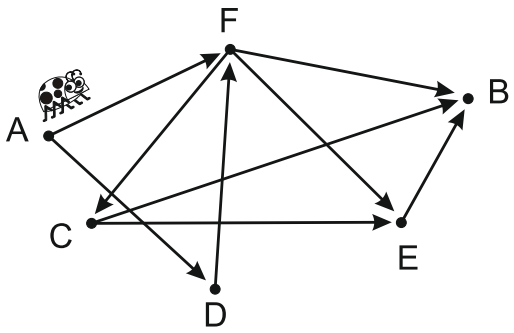
Problem 16. A rope with a length of 91 feet has been cut into 6 equal parts. Find the length of each part in inches. (1 foot is equal to 12 inches)



Problem 17. The numbers 8, 12, 13, 17, 21 and x can be divided into three pairs of numbers with equal sums. What is the number x?

Problem 18. I used 54 points to divide the segment AB into equal segments, each with a length of 12 mm, with none of the points matching A or B. I used 45 points to divide the line segment CD into equal segments, each with a length of 2 cm, with none of the points matching C or D. By how many millimetres is the segment AB shorter than the segment CD?

Problem 19. A ladybird is moving from point A to point B following the direction of the arrows. In how many ways can it get to point B?



Problem 20. John wrote down all three-digit numbers made up of different even digits on some cards. At least how many cards should Peter choose, without looking, in order to be sure that a card with a number divisible by 5 would be among them?

FINAL – AGE GROUP 4

Problem 1. Calculate

$$123456 - 12345 - 1100 - 11$$

Problem 2. Calculate

$$(10 \times 10 \times 10 \times 10 \times 10 \times 10) \div (2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5) - 10 \times 2 \times 5$$

Problem 3. Find the remainder left after dividing 20 222 022 by 9?

Problem 4. Find the ones digit of the number A.

$$A = 41 \times 42 \times 43 \times 51 \times 52 \times 53 \times 2021 - 2021$$

Problem 5. Find x.

$$2 + 4 + 6 + \ldots + 28 + 30 = 1 + 3 + 5 + 7 + \cdots + 29 + x$$

Problem 6. How many three-digit numbers $\overline{abc}$ are there in total, if $a = b \neq c$?

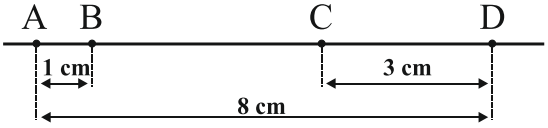
Problem 7. Find the greatest number which, when divided by 111, will have an equal quotient and remainder.

Problem 8. The product of all natural numbers smaller than the natural number n is equal to the product of an odd number and the number 16. Find the possible values of n.

Problem 9. Calculate ■ + □ - ● if 653 - 38□ = ●■4

Problem 10. We are given four consecutive numbers. The product of the first three numbers is 990. Find the product of the last three numbers.

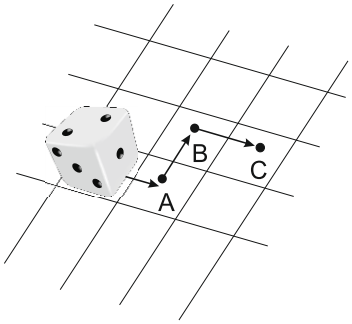
Problem 11. Find the sum of the lengths of all line segments on the drawing below.



Problem 12. The rectangle A has a perimeter of 40 cm. If we increase one of the sides of the rectangle A by 5 cm, we will get a new rectangle B with an area which is 60 cm^2 greater than the area of A. Find the area of rectangle A in cm^2.

Problem 13. Ten points are marked on a segment of length 1 m and 98 cm. None of them coincide with the ends of the segment. The points divide the segment into equal parts. How many centimetres is the greatest distance between two of the marked points?

Problem 14. A die was placed as shown on the picture. Then it rolled on its edge in the direction shown with arrows below. Find the sum of the number of dots on the top face of the die when it is in the squares A, B and C.



Hint: The sum of the number of dots on the opposite faces of the cube is 7.

Problem 15. A square has been divided into identical small squares which have been coloured in black and white in a chess pattern. The number of black squares is 40. Find the number of identical squares that the large square has been divided into.

Problem 16. Find x if

$$\big((x \div 12) \times 9\big) \div 36 - 1 = 1$$

Problem 17. Find the digit x.

$$\underbrace{101999795\ldots x}_{91\text{ digits}}$$

Problem 18. How many different numbers can we get from the number 12345, such that exactly three of the digits would remain in the same place?

Problem 19. The natural numbers from 1 to 199 were written on a board. Peter underlined all numbers divisible by 6, and Kevin underlined all numbers divisible by 9. How many numbers were underlined twice?

Problem 20. Find the smallest natural number with a product of the digits equal to 3600.

MATHEMATICAL RELAY - AGE GROUP 4

*The answer for each problem is represented by the symbols @, #, &, §, **
and must be used to solve each following problem. Each team must fill one answer sheet.

Problem 1. I formed a rectangle using 34 squares, each with a side length of 2 cm. Then I formed another rectangle using another 34 squares, each one also with a side length of 2 cm. The perimeters of both rectangles turned out to be different. The smaller of the two perimeters is @ cm. Find @.

Problem 2. There are # numbers that leave the greatest possible remainder when divided by @. Find #.

Problem 3. The numbers from 1 to # + 17 have been divided into pairs, such that one of the numbers in the pair divides the other number. The greatest possible number of pairs is &. Find &.

Problem 4. The sum of the length and width of a rectangle is equal to & cm. Its side lengths are expressed in integers, in centimetres. The greatest possible value of the area of the rectangle is § cm^2. Find §.

Problem 5. The number equal to §−26 has been expressed as the sum of a few numbers. The greatest possible product of these numbers is 324. The number of addends is *. Find *.

MATHEMATICAL RELAY - AGE GROUP 4

ANSWER KEY

Problem	**1**	**2**	**3**	**4**	**5**
Answer	**76**	**12**	**13**	**42**	**5; 6**

ANSWER KEY - AGE GROUP 4

Round / Problem	AUTUMN	WINTER	SPRING	FINAL
1	111	9 999	0	110 000
2	899	148	7	0
3	5	84	260	3
4	29	67	2	5
5	4	2897	10	15
6	216	3	2	81
7	4	2022	2	12320
8	6	15	72	7 and 8
9	13	4	0	13
10	2 or 3	I	70 or 80	1320
11	16	12	11 667	28
12	40	10	4	96
13	1 or 3	12	7	162
14	1	2	48	14
15	12	108	8	81
16	44	7521	182	96
17	8	36	16 or 4	3
18	6	108, 120	260	10
19	0	20, 24	8	11
20	9876543120	30	37	25589

MATHEMATICS WITHOUT BORDERS

AGE GROUP 5

INSTRUCTIONS

1. Please **DO NOT OPEN** the contest papers until the Exams Officer has given permission.

2. There are 20 open-ended questions in the test. Please write down your answers in the **ANSWER SHEET.**

3. Each problem is scored 2 points for a correct answer; 1 point if there are two or more answers and at least half are given, or if one wrong answer is given in addition to the correct answer; 0 points for a wrong answer or no answer.

4. Working time: not more than 60 minutes. In the case of an equal number of points, the higher ranked participant will be the one who has spent less time solving the problems.

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6. The conditions of the problems contain:

- *Numbers expressed as m/n ($n \neq 0$), where m is a natural number or 0, and n is a natural number*
- *Decimal fractions.*

7. The following must be used for problems with a numerical answer:

- *Numbers expressed as m/n ($n \neq 0$), where m is a natural number or 0, and n is a natural number;*
- *Decimal fractions.*

8. Students are NOT allowed to receive help by the Exams Officer or by anyone else during the contest.

WE WISH YOU ALL SUCCESS!

AUTUMN – AGE GROUP 5

Problem 1. Calculate

$$12345 - 45 - 345 - 2345.$$

Problem 2. Calculate

$$(10 \times 10 \times 10 \times 10 \times 10 \times 10) \div (50 \times 50 \times 50) - 1$$

Problem 3. Find the remainder left after dividing 9 999 987 by 9?

Problem 4. How many tens are there in the number A?

$$A = 25 \times 25 \times 4 \times 8 + 625 \times 16 \times 2$$

Problem 5. How many digits 0 are there in the number A?

$$A = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 999 + 998 + 997 + 996 + 995 + 994 + 993 + 992 + 991$$

Problem 6. Calculate x if the number of minutes in 12 hours is the same as the number of hours in x days.

Задача 7. Let us take a look at the following 50-digit number:

$$\underbrace{2222 \ldots 22}_{50\ digits\ 2}$$

Find the ones digit of the product of the digits of this number.

Problem 8. Each number in the following sequence (after the second number), is equal to the product of the two numbers that precede it. Find the product of the 3 missing numbers.

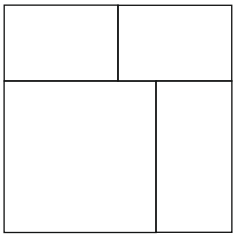
_, _, 12, _, 576

Problem 9. The sum of 11 consecutive numbers is 209. What is the 10^{th} number?

Problem 10. A number is made up of 11 thousands, 111 hundreds, 1111 tens and 11111 ones. What is the number?

Problem 11. The number 10100 can be expressed as the product of two three-digit numbers. Which is the greater number?

Problem 12. A square has been cut into 4 figures with areas of 6 cm^2, 6 cm^2, 8 cm^2 and 16 cm^2. Find the side of the square in cm.



Problem 13. The length of a rectangle is 6 times greater than its width, and its width is 35 cm smaller than its length. Find the perimeter of this rectangle in cm.

Problem 14. The sides of a rectangle are expressed in cm using two even one-digit numbers and two odd one-digit numbers. The area and perimeter of this rectangle are expressed using the same number in cm^2 and cm, respectively. Find the area of this rectangle in cm^2.

Problem 15. Two red points and three blue points are placed along a circle. Ivo counted all possible quadrangles which have exactly one red vertex. It turned out that the number of these quadrangles is smaller than the number of triangles with at least two blue vertices. Find out by how much.

Problem 16. Find the natural numbers that are smaller than 12 and have exactly three factors.

Problem 17. In a box there are 7 red pencils and 8 blue pencils. Let us take some pencils out of the box without looking. At least how many pencils should we take out in order to be sure that among them there would be no fewer than 4 red pencils and no fewer than 3 blue pencils?

Problem 18. The natural two-digit number A has been divided by 14. The remainder left after the division is equal to the quotient. How many possible numbers A are there?

Problem 19. Find the ones digit of the number A.

$$A = 1 \times 7 \times 13 \times \ldots \times 43 \times 49$$

Problem 20. Find the natural number that is greater than 2000 and is such that if we add it to the sum of its digits, we will get 2021.

WINTER – AGE GROUP 5

Problem 1. Calculate

$$(33 \times 333) \div 27 - 7$$

Problem 2. Calculate

$$\frac{1}{3} + 4\frac{2}{3} + \frac{1}{6} + 3\frac{5}{6} + \frac{1}{9} + 2\frac{8}{9}$$

Problem 3. Let

$$A = \underbrace{8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8}_{8} \text{ and } B = \underbrace{16 \times 16 \times 16 \times 16 \times 16 \times 16 \times 16}_{7}$$

Find the denominator of the fraction $\frac{A}{B}$ after reducing it.

Problem 4. The number $\overline{2022a} + 12$ is divisible by 18. Find a.

Problem 5. Calculate $x \div 0.02$, if $x + 0.2 = 1\frac{1}{2} + \frac{4}{10}$.

Problem 6. What is the greatest prime number that is a factor of the greatest number written with different digits, the product of which is equal to 12?

Problem 7. For which natural number n is the following correct?

$$\frac{5-n}{13+n} = \frac{1}{5}$$

Problem 8. Let x and y be even numbers, and let n and m be odd numbers.
If $10 < n < m < x < y < 20$ calculate $x - n$.

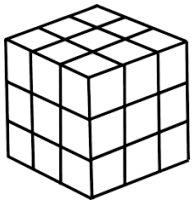
Problem 9. How many natural numbers smaller than 41 are co-prime to the number 41?

Problem 10. Calculate

$$\frac{1}{3} \times \frac{1}{4} \times \frac{1}{5} \times 0.05 \times 12 \times 16 \times 20 - 3$$

Problem 11. One of the natural numbers 1, 2, 3, 4, 6 and x is written on each face of a cube. Peter multiplied the numbers written on the opposite faces of the cube. He got equal products. Find x.

Problem 12. A large cube has been built from 27 identical small cubes painted white. The faces of the large cube have been painted red. How many of the faces of the small cubes have been left white?



Problem 13. I cut out a square with a perimeter of 12 cm out of a square with a side length of 12 cm. What part of the area of the square has been cut? Write down your answer as an irreducible fraction.

Problem 14. The sum of the lengths of two sides of a rectangle is 11 cm and its perimeter is 18 cm. How many cm^2 is the area of the rectangle?

Problem 15. We choose the number *a* among the numbers 2, 7, 12, 17, 22, 27, 32, 37, 42 and 47. We choose the number *b* among the numbers 3, 8, 13, 18, 23, 28, 33, 38, 43 and 48.
How many natural numbers smaller than 50 can be expressed as the sum of the numbers *a* and *b*?

Problem 16. How many different remainders can we get when dividing 2022 by a one-digit number?

Problem 17. How many two-digit natural numbers are there that, when divided by 6 and 7, leave the same remainder?

Problem 18. How many proper fractions with 5 as a denominator are greater than $\frac{5}{11}$ and smaller than $\frac{9}{11}$?

Problem 19. Fewer than 120 trees are growing in an orchard. $\frac{1}{15}$ of them are pear trees, $\frac{7}{12}$ are apple trees, and the rest are plum trees. How many plum trees are there in the orchard?

Problem 20. Find the remainder left after dividing $8! + 5! - 22$ by 2022.
Hint: N! is the product of all natural numbers smaller than or equal to *N*.

SPRING – AGE GROUP 5

Problem 1. Calculate

$$\frac{1}{2}+\frac{1}{3}+\frac{2}{3}+\frac{1}{5}+\frac{4}{5}-\frac{22}{10}.$$

Problem 2. Calculate

$$\frac{1}{2}\times 2.022+\frac{1}{3}\times 2.022-\frac{5}{6}\times 2.022.$$

Problem 3. What is the smallest possible sum of the natural numbers a and b, if

$$\frac{a}{5}<1 \text{ and } \frac{b}{7}>1 ?$$

Problem 4. How many three-digit numbers N are there, for which the numbers $\frac{N}{57}$ and $\frac{N}{12}$ are integers?

Problem 5. The digits 1, 2 and 3 are the only digits used to form all decimals, and in each of them each digit is only used once. How many of these decimals are smaller than 3?

Problem 6. Let $A=2\times 8\times 16\times 32$ and $B=64\times 64\times 64$. Find the denominator of the fraction $\frac{A}{B}$ after reducing it to its lowest terms.

Problem 7. How many 5-digit numbers 3*91* are there, such that are divisible by 22?

Problem 8. How many values of the natural number x are there if x% of 56 is an integer smaller than 100?

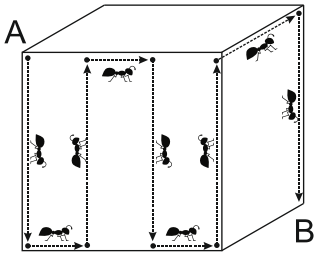
Problem 9. For which natural numbers n, the number equal to $1\times 2\times 3\times ...\times (n-1)\times n$ is divisible by 256 but not by 260?

Problem 10. The natural numbers N, $7\times N$ and $11\times N$ each have exactly three distinct prime factors. What is the smallest value of the number N?

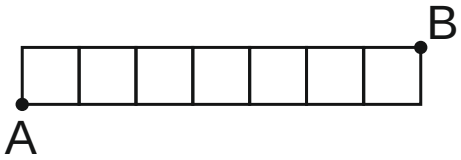
Problem 11. Cyclist A is travelling at 20 metres per second and cyclist B is travelling at 1.3 km per minute. By how many kilometres per hour is cyclist B faster than cyclist A?

Problem 12. One of the diagonals of a quadrilateral with a perimeter of 20.6 cm divides it into two triangles with perimeters of 10.8 cm and 15.7 cm. How many centimeters is this diagonal?

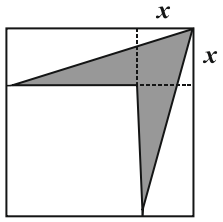
Problem 13. If the edge of the cube is 10 cm, how many cm will the ant travel from point A to point B along the path shown on the drawing?



Problem 14. The side length of each cell is 1 cm. How many different paths with a length of 8 cm can be taken, along the sides of the cells, in order to get from *A* to *B*?



Problem 15. A square with a side length of 12 cm has been divided into two rectangles and two squares. Calculate the area of the shaded part if $x = 5$ cm.



Problem 16. Calculate

$$2022 \div \left(2 \div \left(3 \div \frac{2.022 \times 2022}{20.22 \times 202.2}\right)\right) - 2021$$

Problem 17. What is the ones digit of the smallest number with a product of its digits equal to 2025?

Problem 18. How many primes smaller than 211 are there that have 4 as the sum of their digits?

Problem 19. Find the greatest among the natural numbers a, b and c for which the following is true:

$$\frac{68}{21} = a + \frac{1}{b + \frac{1}{c}}$$

Problem 20. The numbers 1, 9, 17, …, 97, 105 have been written on a white board. The following action is permitted: we can erase two numbers and write down their sum in their place. How many times was this action carried out if there is only one number left on the board?

FINAL – AGE GROUP 5

Problem 1. Calculate

$$\frac{1}{2}+\frac{1}{10}+\frac{4}{20}+\frac{9}{30}+\frac{16}{40}-\frac{5}{10}$$

Problem 2. Calculate x, if

$$\left(1-\frac{1}{4}\right)\times\left(1-\frac{1}{9}\right)\times\left(1-\frac{1}{16}\right)\times\left(1-\frac{1}{25}\right)=\frac{x}{10}$$

Problem 3. What is the smallest possible sum of the natural numbers a and b, if

$$12.5-2.5\times 0.3<a<15<b<28.9\div 1.7$$

Problem 4. What is the greatest possible value of the sum $A+\frac{B}{C}$, if A, B and C are the numbers 2, 4 and 8 in any order?

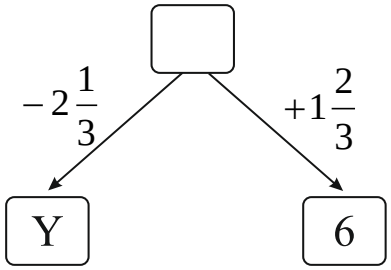
Problem 5. Find $P + Q$, if P is the greater of the fractions $\frac{7}{15}$ and $\frac{7}{16}$, and Q is the smaller of the fractions $\frac{15}{16}$ and $\frac{14}{15}$.

Problem 6. When the natural number x is divided by $1\frac{1}{2}, 2\frac{1}{7}$ and $1\frac{1}{3}$ natural numbers are obtained. Find the smallest possible value of x?

Problem 7. What part of 3.5 hours is equal to 42 seconds?

Problem 8. If Tom works on his own, he will paint Aunt Polly's fence in 4 hours, and if Huck works on his own, he will paint it in 6 hours. If both of them work together for 30 minutes, what part of the fence will remain unpainted?

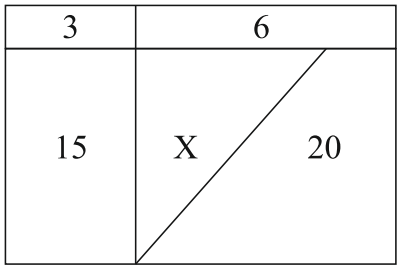
Problem 9. Find Y, if



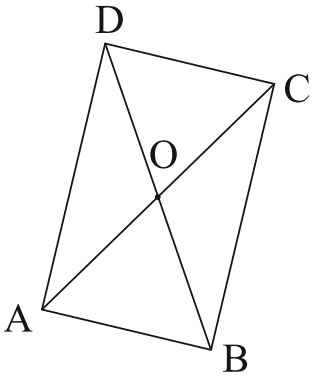
Problem 10. How many irreducible proper fractions are there with one-digit numbers as numerator and denominator, which are greater than $\frac{5}{7}$ and smaller than $\frac{6}{7}$?

$$\frac{5}{7} < \frac{n}{x} < \frac{6}{7}$$

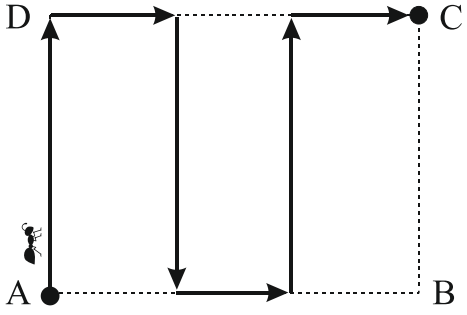
Problem 11. A rectangle has been divided into three rectangles, one trapezium, and one right-angled triangle with areas of 3, 6, 15, 20 and x cm^2, respectively. Find x in cm^2.



Problem 12. The diagonals *AC* and *BD* of the rectangle *ABCD* with an area of 48 cm^2 intersect at point O. The line segments *AO, BO, CO* and *DO* are of equal lengths. The perimeters of the triangles *AOB* and *ABD* are 16 *cm* and 24 cm, respectively. Find the smaller side of the rectangle in *cm*.



Problem 13. An ant travelled the distance from *A* to *C* by following the path marked by arrows. The rectangle *ABCD* has side lengths expressed in integers, in centimetres, and an area of 15 cm^2. How many possible values are there of the distance of the ant's journey?



Problem 14. One of the legs of a right-angled triangle is 25 cm and is 25% greater than the other. Find the area of this triangle in cm^2.

Problem 15. One of the natural numbers 1, 3, 4, 6, 7 and x is written on each face of a cube. Peter added the numbers written on the opposite faces of the cube. He got three equal results. Find x.

Problem 16. For which natural number n is the following correct?

$$\frac{15-n}{13+n}=\frac{1}{3}$$

Problem 17. Let x and y be even numbers, and let n and m be odd numbers.
If $10 < n < x < m < y < 16$ calculate

$$\frac{m+x}{y-n}-\frac{1}{3}$$

Problem 18. If 1 kg and 50 g peaches cost 5.46 USD, how many USD do 800 grams of peaches cost?

Problem 19. The natural number n has two natural factors, and the number $n + 1$ has three natural factors. How many primes are factors of the number equal to $n + 2022$?

Problem 20. We are given the number 1234. How many different numbers can we get if we move the digits around so that exactly one of the digits would remain in the same place?

MATHEMATICAL RELAY - AGE GROUP 5

*The answer for each problem is represented by the symbols @, #, &, §, **
and must be used to solve each following problem. Each team must fill one answer sheet.

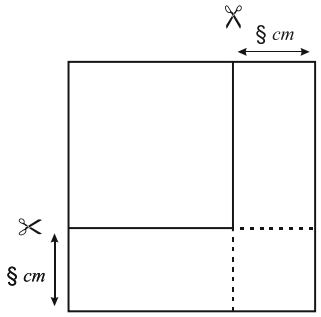
Problem 1. The number N is a natural number smaller than 38. There are $N - 1$ numbers that are smaller than N and co-primes of N. There are @ numbers N. Find @.

Problem 2. Proper fractions with natural numbers as denominator and numerator are written in groups. The fractions which have the same sum of numerator and denominator are grouped together. The groups are arranged in order of the sum of numerator and denominator increasing. Here are the first three groups: $\left\{\frac{1}{1}\right\}, \left\{\frac{1}{2}, \frac{2}{1}\right\}, \left\{\frac{1}{3}, \frac{2}{2}, \frac{3}{1}\right\}, \ldots$
The sum of the fractions in the group which is in $@^{\text{th}}$ place, from left to right, for which both the numerator and the denominator are primes, is #. Find #.

Problem 3. The smallest natural number N, for which $\frac{\#}{N}$ is a proper fraction, is &. Find &.

Problem 4. I used & different digits to write down two natural numbers N and M, such that $N > M$. The smallest possible difference of $N - M$ is §. Find §.

Problem 5. If a square piece of paper is cut as shown on the figure, the remaining part is also a square. Its area is smaller than the area of the original square by 51 cm^2. The side length of the original square is * mm. Find *.



MATHEMATICAL RELAY - AGE GROUP 5

ANSWER KEY

Problem	1	2	3	4	5
Answer	12	125/22	6	3	100

ANSWER KEY - AGE GROUP 5

Round / Problem	AUTUMN	WINTER	SPRING	FINAL
1	9610	400	$\frac{3}{10}$	1
2	7	12	0	6
3	6	16	9	28
4	4000	0	4	10
5	3	85	4	1.4
6	30	23	32	60
7	4	2	5	$\frac{1}{300}$
8	576	3, 5	7	$\frac{19}{24}$
9	23	40	10, 11, 12	2
10	44321	0.2	154	4
11	101	12	6	10
12	6	108	2.95	6
13	98	$\frac{1}{16}$	70	3
14	18	19.25	8	250
15	5	9	35	9
16	4 and 9	3	1012	8
17	12	12	9	8
18	6	2	3	4.16
19	5	21	5	2
20	2014	2000	13	8

MATHEMATICS WITHOUT BORDERS

AGE GROUP 6

INSTRUCTIONS

1. Please **DO NOT OPEN** the contest papers until the Exams Officer has given permission.
2. There are 20 open-ended questions in the test. Please write down your answers in the **ANSWER SHEET.**
3. Each problem is scored 2 points for a correct answer; 1 point if there are two or more answers and at least half are given, or if one wrong answer is given in addition to the correct answer; 0 points for a wrong answer or no answer.
4. Working time: not more than 60 minutes. In the case of an equal number of points, the higher ranked participant will be the one who has spent less time solving the problems.
5. The use of calculators or other electronic devices, as well as books containing formulae is NOT allowed during the course of the contest.
6. The conditions of the problems contain:

   Numbers expressed as a difference of two natural numbers

   Numbers expressed as m/n ($n \neq 0$), where m is an integer, and n is a natural number

   Decimal fractions.
7. The following must be used for problems with a numerical answer:

   Numbers expressed as a difference of two natural numbers

   Numbers expressed as m/n ($n \neq 0$), where m is an integer, and n is a natural number

   Decimal fractions.
8. Students are NOT allowed to receive help by the Exams Officer or by anyone else during the contest.

WE WISH YOU ALL SUCCESS!

AUTUMN – AGE GROUP 6

Problem 1. Calculate

$$\frac{4\times 6+6\times 8+8\times 10+10\times 12}{2\times 3+3\times 4+4\times 5+5\times 6}$$

Problem 2. What number is the reciprocal of A?

$$A=\frac{2}{3}-\frac{1}{6}\times 2$$

Problem 3. Calculate x if $\frac{1}{2}+\frac{1}{3}+\frac{1}{12}+\frac{1}{x}=1$

Problem 4. The difference of two numbers is smaller than the minuend by 12 and greater than the subtrahend by 12. Find the minuend.

Problem 5. What is the smallest three-digit number that, when divided by 4, 14 and 28, leaves a remainder of 2?

Problem 6. Let P be the product of the 30 smallest different prime numbers. Find the remainder when P is divided by 30.

Problem 7. How many of the following fractions are reducible?

$$\frac{1}{84};\frac{3}{84};\frac{5}{84};\ldots;\frac{81}{84};\frac{83}{84}$$

Problem 8. How many integers are there from 9 to 25 that have an even number of natural factors?

Problem 9. If $\frac{1}{11}$ of the number A is equal to 11, calculate 11% of the same number A.

Problem 10. Calculate x if

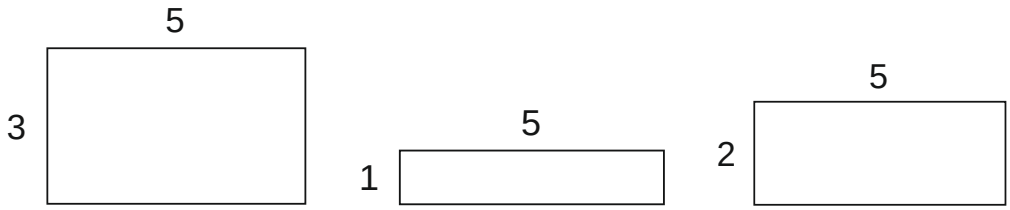
$$0.\overline{51}-\frac{51}{100}=\frac{17}{33}\times x$$

Problem 11. 2 red points and 3 blue points are placed on a circumference. Ivo counted all possible triangles with exactly one red vertex. Their number turned out to be less than the number of triangles with at least two blue vertices. By how much is their number smaller?

Problem 12. A square with a side length of 3 cm and a triangle have equal perimeters and a common side. Find the length of the largest side of the triangle if it is measured in integer centimetres.

Hint: Each side of the triangle is smaller than the sum of the other two sides.

Problem 13. We have three rectangles - one 5 cm long and 3 cm wide, the second 5 cm long and 1 cm wide, the third 5 cm long and 2 cm wide. They have been used to form a rectangle with an area of 30 cm^2. Find all possible values of the perimeter of the resulting rectangle.



Problem 14. What percentage of the figure is blank?



The numbers in the colored rectangles show what part of the large rectangle their areas are.

Problem 15. How many litres of water will fill half of an aquarium in the shape of a rectangular cuboid, which is 50 cm long, 30 cm wide and 40 cm high?

Problem 16. Each of 101 numbers equals either 1 or (-1). Find the total number of possible sums?

Problem 17. Find the natural number smaller than 2000, such that if we add the sum of its digits to it, we would get 2021.

Problem 18. There are 7 red pencils and 9 blue pencils in a box. Let us take some pencils without looking. At least how many pencils should we take to make sure there are no fewer than 4 red pencils and no fewer than 3 blue pencils among them?

Problem 19. Calculate $a + b + c$ if

$$a.bc - 5.9 = 1.62$$

Problem 20. A boat travels downstream from port A to port B in 5 full days. It goes the same distance upstream in 7 full days. How many days will it take for a raft to travel the distance from port A to port B?

WINTER – AGE GROUP 6

Problem 1. Calculate

$$1-2+3-4+5-6+7-\cdots+19-20$$

Problem 2. Calculate

$$(-5)\times(-4)\times(-3)+(-4)\times(-3)\times(-2)+(-3)\times(-2)\times(-1)$$

Problem 3. Calculate

$$(1-9)\times(2-8)\times(3-7)\times\ldots\times(8-2)\times(9-1)$$

Problem 4. Calculate $(-1)^{1+3}+(-1)^{1+3+5+7}+(-1)^{1+3+5+7+9+11}+(-1)^{1+3+5+7+9+11+13}$

Problem 5. Calculate $\frac{1+3^2+3^3+3^5+3^6+3^8}{1+3^3+3^6}$

Problem 6. Find the smallest natural number n for which $n\times 0.\overline{18}-\frac{1}{11}$ is an integer.

Problem 7. Find x, if

$$\frac{14}{19}=\frac{1}{1+\dfrac{1}{2+\dfrac{1}{1+\dfrac{1}{x}}}}.$$

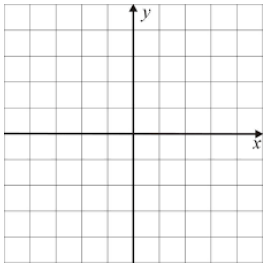
Problem 8. By how much is the sum of negative integers greater than (-10) smaller than the sum of all natural numbers smaller than 10?

Problem 9. Find the greatest three-digit number $\overline{xyz}$ that has different even digits and where

$$x<y;\ y>z;\ z<7.$$

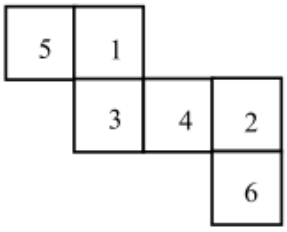
Problem 10. The number 111...111, which is written using 21 ones, is divided by 3. How many digits 3 are there in the quotient?

Problem 11. If $A(2;2)$, $B(1;3)$, C (4,1), find the area of ΔABC.

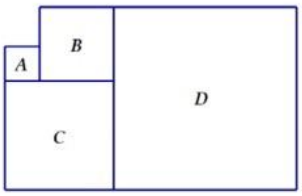


Problem 12. A rectangle that can be divided into three identical squares has an area of 363 cm^2. How many cm is the perimeter of this rectangle?

Problem 13. The following figure shows an unfolded cube. The faces of the cube have been numbered using the numbers from 1 to 6. Find the greatest sum of the numbers that have been written on three faces that share the same vertex.



Problem 14. The figures *A, B, C* and *D* are squares. The perimeter of *A* is half the length of the perimeter of *B*. How many times is the perimeter of *D* greater than the perimeter of *A*?



Problem 15. A rectangular parallelepiped has dimensions of 40 cm, 28 cm and 36 cm. It must be divided into identical cubes with edge lengths expressed in centimeters, in integers. How many such divisions are possible?

Problem 16. Find the number of all natural numbers N, such that N^2 is a factor of 6^6.

Problem 17. A total of 6 digits 0 or 1 are written next to each other, such that the first digit is not 0, and there are no two digits 0 next to each other. How many such six-digit numbers can be written?

Problem 18. The average of 9 numbers is -11 , and the average of 7 of them is -3. Find the average of the other 2 numbers.

Problem 19. How many integers x are there, for which the following inequality is true?

$$-8\pi < x < \pi$$

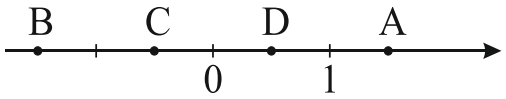
Problem 20. The sum of the minuend, the subtrahend and the difference is 2022. Find the subtrahend.

SPRING – AGE GROUP 6

Problem 1. Calculate

$$\frac{1}{2}-\frac{2}{3}-\frac{4}{3}+\frac{5}{4}-\frac{7}{4}$$

Problem 2. How many of the points A, B, C and D on the figure below correspond to negative numbers on the numerical axis?



Problem 3. Write down the value of the expression as a decimal.

$$2\times10^3+2\times10^1+2\times10^0+3\times10^{-4}$$

Problem 4. Calculate the natural number x if

$$7-3\times3.\bar{1}+|3\times3.\bar{1}-11|=-\frac{x}{3}.$$

Problem 5. Calculate x if

$$48\times2^{11}+5\times2^{16}-7\times2^{15}=3\times2^x$$

Problem 6. Choose two of the numbers – 9, – 8, – 1, 0, 1 and 2, and multiply them. What is the smallest possible result?

Problem 7. All natural numbers smaller than 30 are written on a board. Then all numbers x, for which $30 - x$ is divisible by 7, are erased. How many numbers are left?

Problem 8. Find the ones digit of the number equal to:

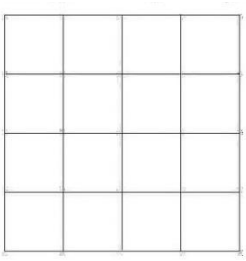
$$15\times25\times35\times45-16\times26\times36\times46$$

Problem 9. How many 5-digit numbers 3*91* which are divisible by 22 are there in total?

Problem 10. Which of the numbers (-49.1567), (-50), $(-\frac{1}{1000})$ and (-0.003) is the greatest?

Problem 11. In how many ways can 38 be expressed as the sum $a + b + c$, where a, b and c are primes and $a < b < c$?

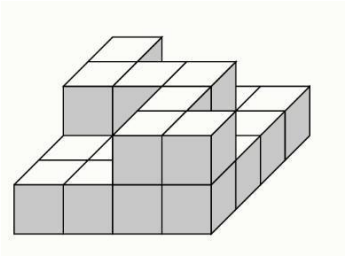
Problem 12. A square is made up of 16 identical squares, each with a side length of 1 cm. If we were to cut the square along a straight line, how many of these 16 squares at most will be cut?



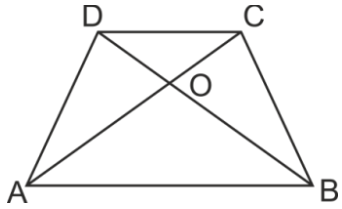
Problem 13. When calculating the area of a circle using the formula πr^2, Alex replaced the length of the radius r with the length of the diameter and got an area of 16π cm^2. By how many cm^2 was Alex wrong?

Problem 14. The surface area of a cube is 726 cm^2. Calculate the volume of this cube in cm^3.

Problem 15. The solid in the figure below consists of 23 identical small cubes. At least how many small cubes do we need to add in order to turn this solid into a large cube?



Problem 16. A trapezium's diagonals have been used to divide it into 4 triangles with areas of 1 cm^2, 3 cm^2, 9 cm^2 and x cm^2. Find x.



Problem 17. Find all integers x, for which the following is true:

$$2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5 = x^2 - 1?$$

Problem 18. What is the sum of the integers a, b and c if the smallest among them is (-15) and $4 \times a = -6 \times b = -15 \times c$?

Problem 19. Find the sum of the digits of the smallest natural number that has 900 as the product of its digits.

Problem 20. The sum of the digits of the two three-digit numbers A and B is 27. Find the smallest possible sum of the digits of the number equal to $A + B$.

FINAL – AGE GROUP 6

Problem 1. Calculate

$$\frac{1}{2}+\frac{1}{10}+\frac{4}{20}+\frac{9}{30}+\frac{16}{40}-\frac{5}{10}$$

Problem 2. Write down the value of the expression as a decimal.

$$2\times 10^3+2\times 10+2\times 10^0+2\times 10^{-1}+2\times\ 10^{-2}+2\times\ 10^{-4}$$

Problem 3. Calculate

$$(2-512)\times(2^2-256)\times(2^3-128)\times\ldots\cdot(256-2^2)\times(512-2)$$

Problem 4. Calculate

$$(-1)^1+(-1)^{1+3+5}+(-1)^{1+3+5+7+9}+\cdots+(-1)^{\overbrace{1+3+\ldots+95+97}^{49}}$$

Problem 5. How many natural numbers are there which are not greater than 21 and can be expressed as the sum of a few different primes?

Problem 6. Find the greatest value of *Z*, if *X* and *Y* are different numbers in the set $\{-2;-1;1;2\}$ and

$$-3\xrightarrow{-X}\mathbf{O}\xrightarrow{\times Y}Z$$

Problem 7. Find the smallest natural number *n* for which

$$n\times 0.\overline{037}-\frac{1}{27}$$

is a natural number.

Problem 8. The sum of the minuend, the subtrahend and the difference is (–204). Find the minuend.

Problem 9. If $3.2<x<4.2$, calculate the value of the expression

$$|x-3.2|+|x-\pi|+|x-4.2|+\pi-x.$$

Problem 10. An ice block is melting evenly by losing a third of its weight per hour. After 3 hours of melting it weighs 27 kg. How much did it weigh at first?

Problem 11. The line segment AB has a length of 120 cm. Annie marked division points on the line segment so that there would be 8 equal parts. Peter marked division points on the line segment so that there would be 12 equal parts. How many of Annie's division points match Peter's division points?

Problem 12. A cuboid has a volume of 12 cm^3. The lengths of the edges are expressed in integers, in centimetres. Find the greatest possible surface area of the cuboid in cm^2.

Problem 13. Three metal cubes with edges of 9 cm, 12 cm and 15 cm, respectively, have been melted in order to form a new cube. Find the edge of the new cube in centimetres.

Problem 14. A cuboid has dimensions of 70 cm, 42 cm and 154 cm. It must be divided into identical cubes with edge lengths expressed in centimeters, in integers. How many such divisions are possible?

Problem 15. The number of line segments that connect each two vertices of an octagonal pyramid is n. Find n.

Problem 16. Calculate

$$\frac{2^{20} + 4^{10} + 16^5}{128^3}$$

Problem 17. Find the smallest value of the natural number n, for which $3^n + 2$ is a composite number.

Problem 18. Find the ones digit of the number equal to

$$15 \times 25 \times 35 \times 45 \times 55 - 16 \times 26 \times 36 \times 46 \times 56 + 1 \times 2 \times 3 \times 4 \times 5$$

Problem 19. Find x, if

$$(-8) + (-7) + (-6) + (-5) + \cdots + 4 + 5 + 6 = (1 + 2 + 3 + 4 + 5) \times x$$

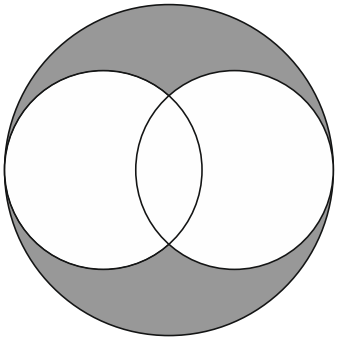
Problem 20. Peter's age is a two-digit number, which is greater than the tripled sum of its digits by 4. How old is Peter?

MATHEMATICAL RELAY – AGE GROUP 6

*The answer for each problem is represented by the symbols @, #, &, § and * and must be used to solve each following problem. Each team must fill one answer sheet.*

Problem 1. Seven numbers have been written on seven cards which have been arranged in a row. The sum of the numbers on each three adjacent cards is (– 6), and the sum of the numbers on all cards is (–7). The number written on the first card is @. Find @.

Problem 2. We are given a circle with a radius of @ cm. Two identical circles with a radius of 3 cm have been inscribed in the large circle, as shown on the drawing below. Let us use X to denote the area of the large circle, and Y to denote the difference of the area of the grey part and the common part of the two white circles. If $\frac{Y}{X} = \#$, find the irreducible proper fraction #.



Problem 3. Let N be a natural number. The number of irreducible proper fractions of the type $\frac{N}{50\times\left(\# + \frac{2}{5}\right) - N}$ is &. Find &.

Problem 4. The number of three-digit numbers smaller than 129 which have exactly & natural factors is §. Find §.

Problem 5. The value of the expression

$$|x - \pi| + |x - 2\pi| + |x - 3\pi| + |x - \pi^2|$$

for x = § is *. Find *.

MATHEMATICAL RELAY – AGE GROUP 6

ANSWER KEY

Problem	**1**	**2**	**3**	**4**	**5**
Answer	5	7/25	8	6	π^2

ANSWER KEY – AGE GROUP 6

Round / Problem	AUTUMN	WINTER	SPRING	FINAL
1	4	-10	-2	1
2	3	-90	2	2022.2202
3	12	0	2022.0003	0
4	36	2	2	-25
5	114	10	16	15
6	0	6	– 18	10
7	18	4	25	28
8	14	90	1	-102
9	13.31	684	5	1
10	0.01	7	$-\frac{1}{1000}$	91.125
11	1	$\frac{1}{2}$	4	3
12	5	88	7	50
13	22 and 26	14	12π	18
14	50	5	1331	4
15	30	3	41	36
16	102	16	3	1.5
17	1996	13	± 8	5
18	13	-39	-1	1
19	14	29	23	-1
20	35	1011	9	25

MATHEMATICS WITHOUT BORDERS

AGE GROUP 7

INSTRUCTIONS

1. Please **DO NOT OPEN** the contest papers until the Exams Officer has given permission.

2. There are 20 open-ended questions in the test. Please write down your answers in the **ANSWER SHEET.**

3. Each problem is scored 2 points for a correct answer; 1 point if there are two or more answers and at least half are given, or if one wrong answer is given in addition to the correct answer; 0 points for a wrong answer or no answer.

4. Working time: not more than 60 minutes. In the case of an equal number of points, the higher ranked participant will be the one who has spent less time solving the problems.

5. The use of calculators or other electronic devices, as well as books containing formulae is NOT allowed during the course of the contest.

6. The conditions of the problems contain:

- *Numbers expressed as a difference of two natural numbers*
- *Numbers expressed as m/n ($n \neq 0$), where m is an integer, and n is a natural number*
- *Decimal fractions.*

7. The following must be used for problems with a numerical answer:

- *Numbers expressed as a difference of two natural numbers*
- *Numbers expressed as m/n ($n \neq 0$), where m is an integer, and n is a natural number*
- *Decimal fractions.*

8. Students are NOT allowed to receive help by the Exams Officer or by anyone else during the contest.

WE WISH YOU ALL SUCCESS!

AUTUMN – AGE GROUP 7

Problem 1. Calculate

$$9 \times 10^4 + 9 \times 10^3 + 11 \times 10^2 - 2021$$

Problem 2. How many integers are there that divide the number A with a remainder of 0?

$$A = 4^2 \times 5 - 3^2 \times 7$$

Problem 3. Calculate x if

$$-\Big(x - \big(2x - (3x - 4)\big)\Big) = 5$$

Problem 4. How many of the symbols '$-$' should we swap with '$+$', so that the following equation would be correct?

$$\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{12} = 1$$

Problem 5. Calculate $x \times y + y \times z + z \times x$ if $x \div y \div z = 3 \div (-4) \div 5$ and $y = -8$

Problem 6. The integer part $[x]$ of a number x is the largest integer that does not exceed x. The fractional part $\{x\}$ of the number x is the number equal to x - $[x]$.

Calculate

$$\left\{-1\frac{1}{3}\right\} + \left\{-\frac{2}{3}\right\} + \left[1\frac{1}{3}\right] + \left[2\frac{2}{3}\right].$$

.

Problem 7. What are the last three digits of the number A?

$$A = 1 \times 4 \times 7 \times 10 \times 13 \times 16 \times 19 \times 22 \times 25$$

Problem 8. Find the quotient of

$$2 + 2^3 + 2^5 + 2^7 + 2^9 + 2^{11}$$

divided by $1 + 2^6$.

Problem 9. Find the greatest value of the expression.

$$\frac{3 + x^2}{0.375 + x^2}$$

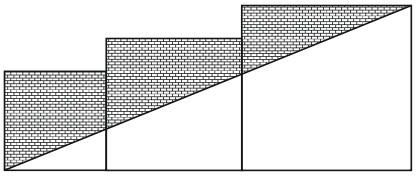
Problem 10. Find the last digit of the number equal to

$$333 \times 666 \times 222 - 222 \times 777 \times 555$$

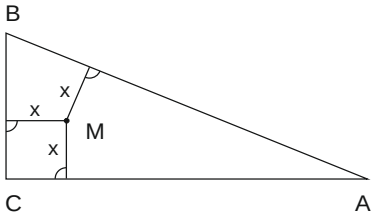
Problem 11. The point $A\ (a + b, a \times b)$ is in quadrant II, and $ab \neq 0$. In which quadrant is the point $B\ (a, b)$?

Problem 12. The perimeters of a rectangle with sides a and b and a square with a side a have a ratio of 5 : 6. Find the ratio of the areas of these figures.

Problem 13. There are three squares with areas of 9 cm^2, 16 cm^2 and 25 cm^2, respectively. Find the area of the shaded figure in cm^2.



Problem 14. The triangle ABC is right-angled with legs AC = 12 cm and BC = 5 cm. The point M is internal to the triangle and is at equal distances x cm from the sides of the right-angled triangle ΔABC. Calculate x.



Problem 15. The sum of the ordinates (x-coordinates) of the points $A(0; 1)$, $B(-2; y)$ and $C(0; -3)$ is equal to the sum of their abscissae (y-coordinates). Find the area of ΔABC.

Problem 16. Points A, B and C lie on a straight line. $AB = 4\ cm$ and $AC = BC + 2\ cm$. Find AC in cm?

Problem 17. If $|a| = 5, |b| = 4$ and $|a + b| = -a - b$, find the value of $a - b$.

Problem 18. What is the total number of three digit numbers $\overline{cab}$ if $c > b > a$ and a, b, c are prime numbers?

Problem 19. A boat travels downstream from port A to port B in 5 full days. It goes the same distance upstream in 7 full days. How many times is the speed of the boat greater than the speed of the river?

Problem 20. A natural number is divisible by all natural numbers x, $x \leq 50$, except two consecutive natural numbers y and $y + 1$, $y \leq 49$. Find y.

WINTER – AGE GROUP 7

Problem 1. Calculate the value of the expression

$$\frac{1}{2}-3\times\left(\frac{1}{6}-4\times\left(\frac{1}{4}-0.75\right)\right)$$

Problem 2. Calculate the value of the expression

$$(x-y\ +\ z)\times(x+y-z)-2x^2+2(y-z)^2,$$

if $x^2-(y-z)^2=1$

Problem 3. Calculate

$$(1-11)\times(2-10)\times(3-9)\times\ldots\times(10-2)\times(11-1)$$

Problem 4. If

$$3^2-5^2+7^2-9^2+11^2-13^2+15^2-17^2=80x,$$

calculate x.

Problem 5. Find the sum of the natural numbers k and n if $kn^2-kn-n^2+n=26$.

Problem 6. The integers x and y are such that $\frac{x}{12}=\frac{y}{8}$ and $xy=96$. Calculate $x+y$.

Problem 7. If $|a|=\ 1-b$ and $|b-2|=4$ calculate a^2-b^2.

Problem 8. Find the number of different prime factors of the number equal to

$$6^{2020}-7\times6^{2021}-6^{2022}.$$

Problem 9. If

$$x^4+2022x^2+2021x+2022=\ (x^2+Ax+1)(x^2-x+B)$$

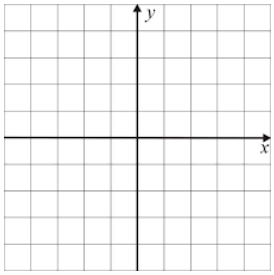
is an identity, calculate $A+B$.

Problem 10. How many natural numbers N are there in total, for which N^3 is a factor of 12^6?

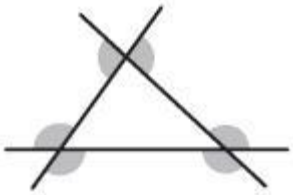
Problem 11. The base of a pyramid with 2022 vertices is also the base of a prism. How many edges does the prism have?

Problem 12. A new triangle is formed from a right-angled triangle with legs of 6 cm and 8 cm and a right-angled triangle with a hypotenuse of 17 cm and a leg of 15 cm. How many centimeters is the perimeter of the new triangle?

Problem 13. If $A(2; 2)$, $B(1; 3)$, C (4,1), calculate the area of $\Delta\, ABC$.



Problem 14. How many degrees is the sum of the angles colored in gray?



Problem 15. The quadrilateral *ABCD* is a rectangle. The bisector of $\measuredangle BAD$ intersects the straight lines CB and CD at the points N and M respectively, and DM = 5 cm, CN = 3 cm. Calculate the area of the rectangle.

Problem 16. One of the digits of the three-digit number $\overline{abc}$ is 0. When 0 is removed, we get a two-digit number that is 9 times smaller. What is the three-digit number?

Problem 17. Each participant in a chess tournament plays one game against each of the other participants. For each game the winner gets 1 point, in case of a draw the participants get 0.5 points, in case of a loss – 0 points. The winner of the tournament won all his games and received a quarter of the points equal to the sum of points won by the other participants. How many chess players participated in this tournament?

Problem 18. Calculate

$$|\pi - 1| + |\pi - 2| + |\pi - 3| + |\pi - 4| + |\pi - 5| + |\pi - 6|$$

Problem 19. Calculate $a + b + c$ if

$$4a^2 + 9b^2 + 36c^2 = 4a + 6b + 12c - 3.$$

Problem 20. What are the numbers x for which the following equation is true?

$$\big||x^3| - 3x^3\big| = -x$$

SPRING – AGE GROUP 7

Problem 1. Calculate

$$0.1^4 - (\frac{1}{10} - 1) \times (1 + \frac{1}{10}) \times (1 + \frac{1}{100})$$

Problem 2. Calculate x if

$$(-x - \pi) \times |\pi - 4| = (4 - \pi)^2$$

Hint: $\pi \approx 3.14$

Problem 3. Write the value of the following expression as a decimal.

$$(10^3 + 2 \times 10^1 + 3 \times 10^0 + 4 \times 10^{-2}) \div 10230.4$$

Problem 4. Calculate $x - 100$, if $\frac{102^3 - 8}{100} + 204 = x^2$

Problem 5. The number A is the product of all primes smaller than 22. The number B is the product of all even numbers smaller than 22. Find the ones digit of the number equal to $A - B$.

Problem 6. Find the smallest prime factor of the number equal to

$$15^3 + 17^3 + 19^3 + 24^3.$$

Problem 7. Find x, if

$$10 \times (11^5 + 11^4 + 11^3 + 11^2 + 12) + 1 = 121^x.$$

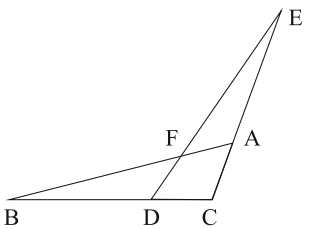
Problem 8. Find x, if $x^3 - x = 3 - 3x$.

Problem 9. If the natural numbers a and b, when divided by 5, leave remainders of 1 and 2, respectively, find the remainder left after dividing the expression $a + b^2$ by 5.

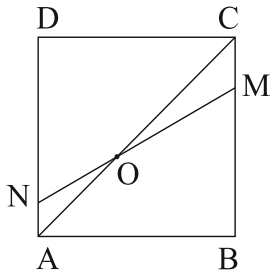
Problem 10. What is the smallest natural number n for which $n^3 + 5n - 1$ is not a prime number?

Problem 11. Find the area of a right-angled triangle with side lengths of x *cm*, 252 *cm* and 277 cm, if $x < 252 < 277$.

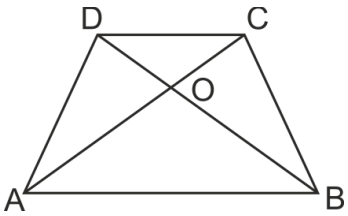
Problem 12. Two identical triangles - *ABC* and *DEC*, have been drawn as shown on the figure below. If *DC* = *AC* = 1 cm, *BD* = *EA* = 2 cm, and the area of quadrilateral $DCAF = 1\ cm^2$, find the area of ΔABC.



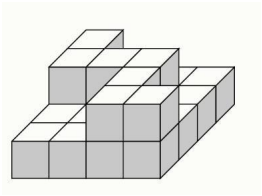
Problem 13. Find the measure of $\measuredangle COM$ if $\measuredangle OND = 60°$ and the quadrilateral $ABCD$ is a square.



Problem 14. A trapezium's diagonals have been used to divide it into 4 triangles with areas of 1 cm^2, 3 cm^2, 9 cm^2 and x cm^2. Find x.



Problem 15. The solid on the figure below consists of 23 identical small cubes. At least how many small cubes do we need to add in order to turn this solid into a cuboid?



Problem 16. The length of a circle with a diameter $\frac{8}{\pi}$ cm is equal to the perimeter of a square. Find the area of this square in cm^2.

Problem 17. The even natural number N has exactly 7 natural factors (including 1 and N). How many natural factors does the number $12 \times N$ have?

Problem 18. How many integers are there from 11 to 1000 where no two consecutive digits are the same?

Problem 19. The expression $n^3 + 7n^2 - 36$ is factored into three factors. Two of its factors are $(n-2)$ and $(n+3)$. Which is the third?

Problem 20. We are given a sequence made up of pairs of numbers, out of which we must remove one pair. (1, 2), (3, 4), …,$((x-1), x)$ The sum of the numbers which have not been removed is 156. Calculate the sum of the removed numbers.

FINAL – AGE GROUP 7

Problem 1. Calculate

$$1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\frac{1}{16}-\frac{1}{32}$$

Problem 2. Calculate

$$0.00000256+\left(1-\frac{1}{5}\right)\times\left(1+\frac{1}{5}\right)\times(1+\frac{1}{25})\times(1+\frac{1}{625})$$

Problem 3. Calculate the sum of all negative integers x, for which

$$(-x-\pi)\times|\pi-4|=(\pi-4)^2$$

Hint: $\pi\approx 3.14$

Problem 4. The integer part $[x]$ of a number x is the greatest integer that is not greater than x. The fractional part $\{x\}$ of the number x is the number equal to $x-[x]$.

Calculate $\{-\pi\}+\{-2\pi\}+[\pi]+[2\pi]$

Problem 5. If $(x-y)xy\neq 0$ and $\frac{x^3-y^3}{(x-y)^3}=7$, calculate $\frac{x}{y}+\frac{y}{x}$.

Problem 6. The number 2 is a root of the equation $|x-5|=a^2-3a+5$, where a is a parameter. Find the other root.

Problem 7. When dividing the natural number A by 55 we get a remainder of 34. When dividing the same number by 32, we get a remainder of 2. Find the ones digit of the number A.

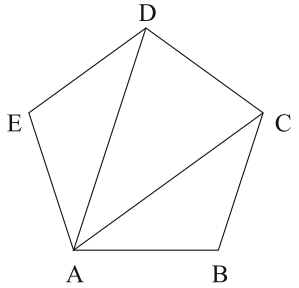
Problem 8. Find the remainder left after dividing $2^{22}-6^{12}$ by 10.

Problem 9. Find x, if

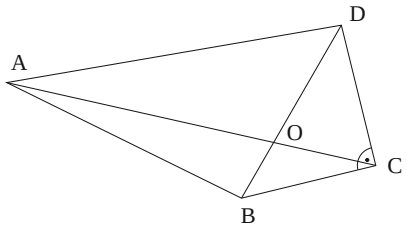
$$|x|-\left|\frac{x}{-6}\right|=x-\frac{1}{3}\times(\frac{x}{2}-1)$$

Problem 10. Find the sum of the natural numbers m and n, for which $20m+22n=222$.

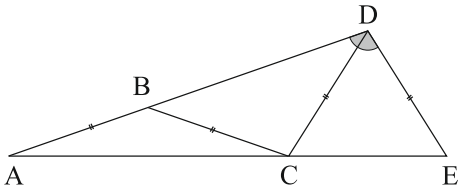
Problem 11. We are given a regular pentagon $ABCDE$. Measure the angle $\measuredangle CAD$.



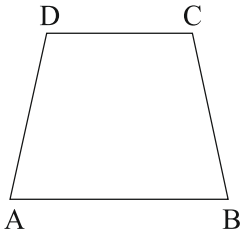
Problem 12. We are given the quadrangle *ABCD*. It is known that *DB* = 13 *cm*, *BC* = 5 *cm*, $\measuredangle BCD = 90°$, *AO* : *AC* = 2:3. Calculate its area in cm^2.



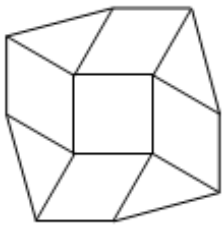
Problem 13. On the drawing below, we are given *AB* = *BC* = *CD* = *DE*. The measure of the angle $\measuredangle ADE$ is expressed in integer degrees. Find the smallest possible value of the measure of $\measuredangle ADE$.



Problem 14. Each of the diagonals of the trapezium below is equal to the sum of the two bases. How many degrees is the angle between the diagonals?



Problem 15. The figure on the drawing is made up of one square in the centre, four rhombuses with equal acute angles and four triangles. Find the area of each of the rhombuses in cm^2, if the area of the square is $36\ cm^2$, and the area of the entire figure is $204\ cm^2$.



Problem 16. The number n is a natural number, and the number $m = 4n^2 - 121$ is a prime. Find m.

Problem 17. Find ab if $a + b = 3$ and $a^3 + b^3 = 63$.

Problem 18. How many natural numbers N are there, such that among the numbers from 1 to N exactly 12% are divisible by 8?

Problem 19. We have scales with two pans and three weights of 1 kg, 3 kg and 10 kg, respectively. How many items of different weights can we weigh on the scales if we only weigh one item at a time?

Problem 20. Let x and y be primes, such that $2y + 1$ is divisible by x, and $2x + 1$ is divisible by y. Find the smallest possible value of $x + y$.

MATHEMATICAL RELAY – AGE GROUP 7

*The answer for each problem is represented by the symbols @, #, &, §, * and must be used to solve each following problem. Each team must fill one answer sheet.*

Problem 1. The sum of the roots of the equation

$$4x^2 - 12x + 9 - 5|4x - 6| + 25 = 0$$

is @. Find @.

Problem 2. I added all two-digit numbers which are divisible by @. The greatest prime that is a factor of the resulting sum is #. Find #.

Problem 3. Two of the sides of a right-angled triangle are # and $\# - 2$, and the third side is also an integer. The area of the triangle is &. Find &.

Problem 4. § points were marked inside a rectangle. The rectangle was then divided into triangles with vertices that either match these points and/or the vertices of the rectangle. The result was & triangles. Find §.

Problem 5. Let us observe all rectangles with a perimeter of § cm and side lengths expressed in primes, in centimetres. We must calculate their areas and subtract the smallest area from the greatest area. We will get * cm^2. Find *.

MATHEMATICAL RELAY - ANSWER KEY

Problem	**1**	**2**	**3**	**4**	**5**
Answer	3	37	210	104	432

ANSWER KEY – AGE GROUP 7

Round / Problem	AUTUMN	WINTER	SPRING	FINAL
1	98079	-6	1	$\frac{21}{32}$
2	4	-1	-4	1
3	-0.5	0	0.1	-6
4	1	-2	-20 and 4	$-3\pi + 20$
5	-68	16	0	2.5
6	4	± 20	3	8
7	000	5	3	4
8	42	4	1	8
9	8	2023	0	$-\frac{1}{5}$
10	4	15	6	11
11	III	6063	14490	36
12	2:3	48	2	90
13	20	$\frac{1}{2}$	$15°$	64
14	2	720	3	$60°$
15	4	10, 40	9	24
16	3	405	4	23
17	$-1; -9$	10	18	-4
18	4	9	809	7
19	6	1	$n + 6$	13
20	31	$0; -\frac{1}{2}$	15	10

MATHEMATICS WITHOUT BORDERS
AGE GROUP 8

INSTRUCTIONS

1. Please **DO NOT OPEN** the contest papers until the Exams Officer has given permission.

2. There are 20 open-ended questions in the test. Please write down your answers in the **ANSWER SHEET.**

3. Each problem is scored 2 points for a correct answer; 1 point if there are two or more answers and at least half are given, or if one wrong answer is given in addition to the correct answer; 0 points for a wrong answer or no answer.

4. Working time: not more than 60 minutes. In the case of an equal number of points, the higher ranked participant will be the one who has spent less time solving the problems.

5. The use of calculators or other electronic devices, as well as books containing formulae is NOT allowed during the course of the contest.

6. The conditions of the problems contain both rational and irrational numbers.

8. Both rational and irrational numbers must be used for problems with a numerical answer.

9. Students are NOT allowed to receive help by the Exams Officer or by anyone else during the contest.

WE WISH YOU ALL SUCCESS!

AUTUMN – AGE GROUP 8

Problem 1. Calculate

$$\frac{2^{24}+4^{12}+8^{8}+16^{6}}{2^{26}}$$

Problem 2. Calculate the value of the expression: $\left(\frac{|x|}{x}+\frac{|y|}{y}\right)^{3}-8$ if $xy>0$.

Problem 3. If $x<2$ and

$$\frac{1}{x}=\frac{x^{3}}{1+3+5+7+9+11+13+15+17},$$

find x.

Problem 4. The following equations are true for the integers x, y and z:

$$x\times y=-2;\ \ y\times z=-6.$$

Calculate the greatest possible sum of these integers.

Problem 5. Calculate $x\times y$ if $4y-y^{2}=x^{2}-6x+13$.

Problem 6. Find the number A if the following equation is an identity.

$$(x-3)\times(x-1)\times(x+5)\times(x+7)=(x^{2}-A\times x-13)^{2}-64$$

Problem 7. What is the greatest prime factor of the number equal to A?

$$A=1+3\ +5\ +\cdots+63+65$$

Problem 8. Calculate

$$\left(\frac{1}{1}-\frac{1}{17}\right)\times\left(\frac{1}{3}-\frac{1}{15}\right)\times\left(\frac{1}{5}-\frac{1}{13}\right)\times\ldots\times\left(\frac{1}{15}-\frac{1}{3}\right)\times\left(\frac{1}{17}-\frac{1}{1}\right).$$

Problem 9. If p, q and r are primes, such that $r=99+p=40+q$, calculate $p+q+r$.

Problem 10. Calculate x, if

$$(x+1)(2x^{2}+1)(6x^{2}+2)=3(x+1)(4x^{2}+2)(3x^{2}+1).$$

Problem 11. If the sum of all the angles of the faces and base of a pyramid is 2160 degrees, how many vertices does the pyramid have?

Problem 12. Let the abscissa (x-coordinate) and the ordinate (y-coordinate) of point A be x and y, respectively, and let $x - y = xy = \frac{x}{y}$.

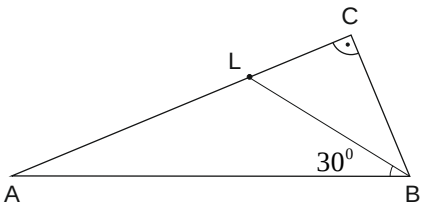
Which quadrant does point A belong to?

Problem 13. What is the ratio of the area of a regular hexagon with a side length of 1 cm and the area of an equilateral triangle with a side length of 2 cm?

Problem 14. The area and perimeter of a rectangle are S cm^2 and P cm, respectively. We reduce both the length and the width of this rectangle by 2 cm. How many cm^2 is the area of the resulting rectangle?

Problem 15. How many triangles are there that have for coordinates of their vertices natural numbers less than or equal to 3?

Problem 16. In the figure, $\measuredangle ACB = 90°$, BL is the bisector of $\measuredangle ABC$ and $\measuredangle ABL = 30°$.



If AC = 24 cm, how many centimeters is the length of the segment BL?

Problem 17. Simplify

$$\frac{(b-c)^3 + (c-a)^3 + (a-b)^3}{(a-b) \times (b-c)}$$

Problem 18. Which of the numbers a, b and c is the smallest?

$$a = 2^{56}, b = 18^{15}, c = 30^{11}$$

Problem 19. How many non-negative integers x_1, x_2, x_3 with a sum of 4 are there?

Hint: The number of non-negative integers $x_1, x_2, \ldots, x_n$ that are solutions of the equation $x_1 + x_2 + \cdots + x_n = N$ is equal to C_{N+n-1}^{n-1}.

Problem 20. If x and y are natural numbers such that $xy + 2x + y = 4$, calculate $x + 2y$.

WINTER– AGE GROUP 8

Problem 1. Calculate the value of the expression

$$(\sqrt{6})^2 + (\sqrt{4})^{2+4} + (\sqrt{2})^{2+4+6}$$

Problem 2. Calculate the value of the expression

$$(-a + b + c)(a - b + c) - ab,$$

if $a^2 + b^2 = c^2$ and $ab = 12$.

Problem 3. Calculate

$$\sqrt{50 \times 72 \times 98 \times 128} - 2022$$

Problem 4. Calculate $\sqrt{\pi^2 - 4\pi + 4} + \sqrt{\pi^2 - 8\pi + 16}$

Problem 5. Find the greatest common factor of numbers equal to $2n + 23$ and $n + 15$ if n is a natural number.

Problem 6. Let a be a natural number, such that the equation $(x + a)(x + 1) + 2 = 0$ has two different real roots. Find the smallest value of a.

Problem 7. Calculate x, if $2x + 3y + 6z = -1.5$ and $3x + 4y + 8z = 5$.

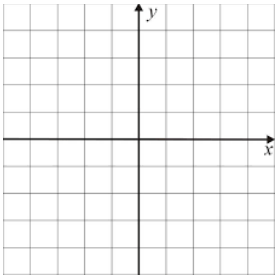
Problem 8. The sum of 100 different natural numbers is equal to 5051. Find the greatest among these numbers.

Problem 9. Find the natural number n, if $\sqrt{13 - \sqrt{48}} = n\sqrt{3} - 1$.

Problem 10. Find the smallest natural number n for which the following inequality is true.

$$(\sqrt{2} + 1)^n - (\sqrt{2} - 1)^n > 10$$

Problem 11. If the points A (2; 1) and B (0; 3) are plotted on the following rectangular coordinate system, find the coordinates of the point M, which is a midpoint of the line segment AB.



Problem 12. The height CH of the isosceles trapezium $ABCD$ ($H \in AB$) has a length of 12 cm. If AC = 13 cm, find the area of the trapezium in cm^2.

Problem 13. Let a, b and c be the sides of a triangle with an area of 8 cm^2. If $a \leq b \leq c$, find the smallest possible value of b in centimeters.

Problem 14. A rectangular parallelepiped has dimensions of 60 cm, 84 cm and 96 cm. It must be divided into identical cubes with edge lengths expressed in integers, in centimetres. How many such divisions are possible?

Problem 15. An N-gon ($N \geq 4$) has $(7N - 26)$ diagonals. Find N.

Problem 16. One of the sides of a triangle is 10 cm and the medians to the other two sides are equal to 9 cm and 12 cm, respectively. Find the area of the triangle.

Problem 17. If a and b are integers, how many different remainders can we get when dividing $a^2 + 3b^2$ by 8?

Problem 18. Calculate $A + B + D$ if

$$\frac{n^4 + n^2 + 1}{n^2 - n + 1} = An^2 + Bn + C + \frac{Dn + E}{n^2 - n + 1}$$

Problem 19. All possible three-digit numbers made up of different digits are written using the digits 0, 1, 2 and 3. What is the probability that a number, chosen randomly among these would be even?

Problem 20. If the two-digit number $\overline{ab}$ is a multiple of 9 and the number $\overline{ab0ab}$ has 6 different prime factors, what is the number $\overline{ab}$?

SPRING – AGE GROUP 8

Problem 1. Calculate the value of the expression

$$(\sqrt{3}-2)^{-1}+(\sqrt{3}-2)^{0}+(\sqrt{3}-2)^{1}$$

Problem 2. If x_1 and x_2 are roots of the equation $x^2-2x-195=0$ and $x_1<x_2$, calculate

$$x_1+2x_2.$$

Problem 3. Calculate m, if $3x^{m^2}+2x^{m+2}+x^4$ is a monomial (only has 1 term).

Problem 4. If $n=\sqrt{7}$, calculate

$$\frac{\sqrt{(n-2)\times(n-1)\times(n+1)\times(n+2)}}{3}$$

Problem 5. Find the greatest common factor of the numbers equal to $3n$ + 23 and n + 7, if n is a natural number.

Problem 6. If $a+2b=5,\ 5b+4c=22$ and $3c+6a=15$, calculate $a+b+c$.

Problem 7. For how many integers n is the value of $(n^2-2n+2)(n^2+2n+2)$ a prime number?

Problem 8. If $\overline{xy}=(x+y)^2, x\neq 0$ calculate $\sqrt{\frac{\overline{yx}}{2}}$. ($\overline{xy}$ is a two-digit number)

Problem 9. If $x\neq -1$, calculate

$$\frac{x^5+x^4+x^3+x^2+x+1}{(x+1)(x^4+x^2+1)}-2.$$

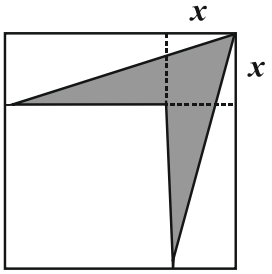
Problem 10. Calculate

$$\frac{4^n}{4^n+2}+\frac{4^{1-n}}{4^{1-n}+2}.$$

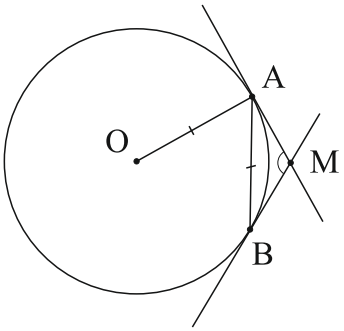
Problem 11. Find the area of a right-angled triangle with side lengths of

x *cm*, $(x+1)$ *cm* and $(x+9)$ *cm*.

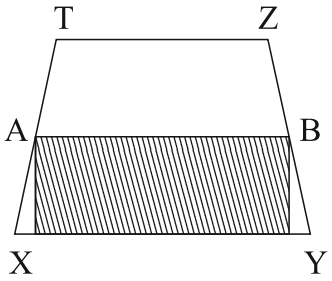
Problem 12. A square has been divided into two rectangles and two squares, one with a side length of 6 cm, and the other with a side length of x cm. Use x to express the area of the gray area.



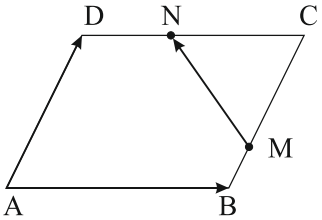
Problem 13. If $OA = AB$, calculate $\measuredangle AMB$ in degrees.



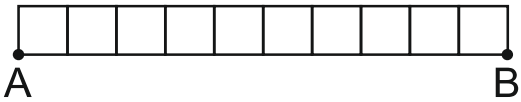
Problem 14. On the drawing below, A and B are midpoints of the legs of the trapezium $XYZT (XY \parallel ZT, XY > ZT)$. The area of the shaded rectangle is 14 cm^2. Find the area of the trapezium $XYZT$ in cm^2.



Problem 15. The quadrilateral *ABCD* is a parallelogram. The points M and N are on the sides BC and CD, respectively, and are such that DN : NC = 2 : 3 and BM : MC = 1: 4. Calculate $x + y$, if $\overrightarrow{MN} = x\overrightarrow{AB} + y\overrightarrow{AD}$.



Problem 16. The side length of each of the cells is 1 cm. How many different routes with a length of 12 cm along the sides of the cells can we take in order to get from *A* to *B*?



Problem 17. John wrote down a natural number using different digits with a product of 36. How many such different natural numbers can be written down in total?

Problem 18. Find the smallest natural number x for which the following inequality is true.

$$(x-1)(x^2-4x+3) > 0$$

Problem 19. If a and b are integers, how many different remainders can we get when dividing $a^2 + b^2$ by 4?

Problem 20. Find the two-digit number $\overline{ab}$ if it is a multiple of 9 and has 6 different factors.

FINAL – AGE GROUP 8

Problem 1. Calculate

$$\left(\left(1-\sqrt{2}\right)^{-1}+\left(1+\sqrt{2}\right)^{-1}\right)^{-1}$$

Problem 2. For how many one-digit natural numbers N is the number equal to

$$\sqrt{4+5N}-\sqrt{7+2N}$$

a rational number?

Problem 3. Find y, if $a-x^2+x\neq 0$ and

$$x^3-(a+1)x^2+a^2=(a-x^2+x)(a-x+1+y)$$

Problem 4. If x_1 and x_2 are roots of the equation $x^2-4x-621=0$ and $x_1<x_2$, calculate

$$x_1-x_2.$$

Problem 5. Find m, if $x^{m^2}+(m+2)x^6+x^4$ is a monomial (only has 1 term).

Problem 6. The integer part $[x]$ of a number x is the greatest integer that is not greater than x. Calculate $[\sqrt{2}+\sqrt{5}+\sqrt{6}]$.

Problem 7. Find the natural numbers N, such that among the numbers from 1 to N exactly 15% are divisible by 6.

Problem 8. Calculate

$$\sqrt{1+(3-a)\times\sqrt{1+(2-a)\times\sqrt{1+(a-1)\times(a+1)}}}$$

if $a<0$.

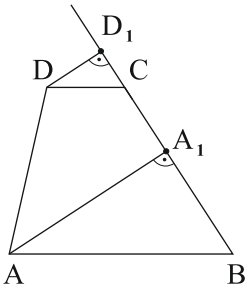
Problem 9. The number n is a natural number and the number equal to n^5+7n has exactly 4 natural factors. Find all possible values of n.

Problem 10. Which of the numbers a, b and c is the smallest?

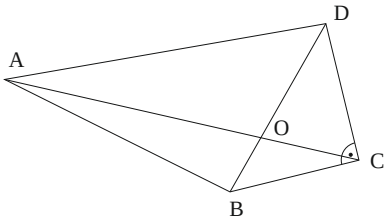
$$a=\sqrt{11}-\sqrt{10};\ \ b=\sqrt{12}-\sqrt{11}\ ;\ \ c=\sqrt{13}-\sqrt{12}$$

Problem 11. The quadrangle $ABCD$ is a trapezium with an area of $25\ cm^2$ ($AB \parallel CD, AB > CD$), $AA_1 \perp BC, A_1 \in BC,\ DD_1 \perp BC, D_1 \in BC, BC = 5\ cm, AA_1 = 7\ cm$.

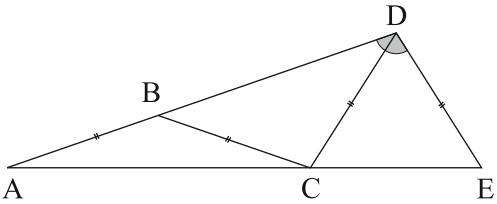
Find the length of the line segment DD_1 in cm.



Problem 12. We are given the quadrangle *ABCD*. It is known that *DB* = 113 *cm*, *BC* = 112 *cm*, $\measuredangle BCD = 90°$ and *AO* : *OC* = 2 : 1. Find its area in cm^2.

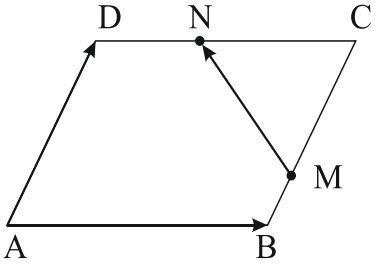


Problem 13. On the diagram below *AB* = *BC* = *CD* = *DE*. The measures of the angles on the diagram with vertices matching the points *A*, *B*, *C*, *D* and *E* are expressed in integer degrees. Find the smallest possible degree value of $\measuredangle ADE$.

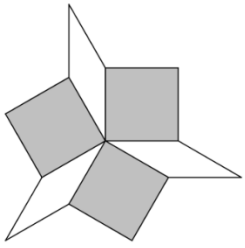


Problem 14. The quadrilateral *ABCD* is a parallelogram. The points *M* and *N* are on the sides *BC* and *CD*, respectively, and are such that *DN* : *NC* = 1 : 2 and *BM* : *MC* = 1: 3.

Find $x + y$, if $\overrightarrow{MN} = x\overrightarrow{AB} + y\overrightarrow{AD}$.



Problem 15. The figure below is composed of three squares and three identical white rhombuses.



The perimeter of the figure is 120 cm. Find the area of the figure.

Problem 16. If x and y are natural numbers such that $xy + 3x + y = 5$, find $x + y$.

Problem 17. How many of all the products of two different natural two-digit numbers are divisible by 5?

Problem 18. Find the ones digit of the number equal to

$$(10^2 - 9^2) \times (9^2 - 8^2) \times \ldots \times (2^2 - 1^2) \times (1^2 - 0^2)$$

Problem 19. If $ab = a + 2b, ab \neq 0$ calculate

$$\frac{a}{b} + 4 \times \frac{b}{a} - ab$$

Problem 20. Let x and y be natural numbers, such that $\sqrt{2xy^2} + \sqrt{3xy}$ is a natural number. Find the smallest value of $x + y$.

MATHEMATICAL RELAY – AGE GROUP 8

The answer for each problem is represented by the symbols ***@, #, &, §, ****
and must be used to solve each following problem. Each team must fill one answer sheet.

Problem 1. The greatest 4-digit number that can be a discriminant of a quadratic equation with integer coefficients is @. Find @.

Problem 2. The number of different digits in the number equal to $(@ + 2)^3$ is #. Find #.

Problem 3. We have # types of flowers. We can use those to make bouquets of 5 flowers each in & ways. Find &.

Problem 4. The lengths of the edges of a cuboid are expressed in natural numbers, in centimetres. The areas of two of its faces are & cm^2 and 14 cm^2, respectively. The volume of the cuboid is V cm^3. The smallest value of V is §. Find §.

Problem 5. If

$$A = \sqrt{§ + \sqrt{§ + \sqrt{§ + \sqrt{§}}}},$$

the greatest integer which is not greater than A is *. Find *.

MATHEMATICAL RELAY – ANSWER KEY

Problem	**1**	**2**	**3**	**4**	**5**
Answer	9997	4	56	56	7

ANSWER KEY – AGE GROUP 8

Round / Problem	AUTUMN	WINTER	SPRING	FINAL
1	1	134	-3	$-\frac{1}{2}$
2	0 or -16	12	17	2
3	-3	4698	2	-1
4	7	2	$\sqrt{2}$	-50
5	6	1 or 7	1 or 2	-2
6	-4	4	6	6
7	11	21	2	20; 40
8	0	101	3	$2-a$
9	164	2	-1	1; 2
10	-1	3	1	c
11	8	M (1;2)	210	3
12	3	60	$6x$	2520
13	3 : 2	4	$120°$	64
14	$S-P+4$	6	28	$\frac{1}{12}$
15	76	4, 13	$\frac{1}{5}$	450
16	16	72	55	2
17	$3(c-a)$	6	38	1449
18	c	2	4	5
19	15	$\frac{5}{9}$	3	-4
20	3	90	18, 45, 63, 99	8

MATHEMATICS WITHOUT BORDERS
AGE GROUP 9

INSTRUCTIONS

1. Please **DO NOT OPEN** the contest papers until the Exams Officer has given permission.

2. There are 20 open-ended questions in the test. Please write down your answers in the **ANSWER SHEET.**

3. Each problem is scored 2 points for a correct answer; 1 point if there are two or more answers and at least half are given, or if one wrong answer is given in addition to the correct answer; 0 points for a wrong answer or no answer.

4. Working time: not more than 60 minutes. In the case of an equal number of points, the higher ranked participant will be the one who has spent less time solving the problems.

5. The use of calculators or other electronic devices, as well as books containing formulae is NOT allowed during the course of the contest.

6. The conditions of the problems contain both rational and irrational numbers.

7. Both rational and irrational numbers must be used for problems with a numerical answer.

8. Students are NOT allowed to receive help by the Exams Officer or by anyone else during the contest.

WE WISH YOU ALL SUCCESS!

AUTUMN – AGE GROUP 9

Problem 1. Calculate

$$\frac{x^3}{y^3} + 2 \times \frac{x^2}{y^2} + 3$$

if $|x| = |y|$ and $y \neq 0$.

Problem 2. One of the factors of the expression $x^4 + 3x^2 + 4$ is $x^2 + x + 2$. Find the other factor.

Problem 3. How many real roots does the following equation have?

$$4x^4 - 3x^3 - 2 = 0$$

In mathematics, Descartes' rule of signs, first described by René Descartes in his work La Géométrie, is a technique for getting information on the number of positive real roots of a polynomial. It asserts that the number of positive roots is at most the number of sign changes in the sequence of polynomial's coefficients (omitting the zero coefficients), and that the difference between these two numbers is always even. This implies, in particular, that if the number of sign changes is zero or one, then there are exactly zero or one positive roots, respectively.

By a homographic transformation of the variable, one may use Descartes' rule of signs for getting a similar information on the number of roots in any interval. This is the basic idea of Budan's theorem and Budan–Fourier theorem. By repeating the division of an interval into two intervals, one gets eventually a list of disjoint intervals containing together all real roots of the polynomial, and containing each exactly one real root. Descartes rule of signs and homographic transformations of the variable are, nowadays, the basis of the fastest algorithms for computer computation of real roots of polynomials (see Real-root isolation).

Descartes himself used the transformation $x \to -x$ for using his rule for getting information of the number of negative roots.

Problem 4. If $a \neq b$ and $x^2 - x + 1 = (x - a)(x - b)$ is true for each value of x, find

$$\frac{a}{a-1} + \frac{b}{b-1}$$

Problem 5. Find the remainder left when 1234567890123456789 is divided by 15.

Problem 6. Which of the numbers a, b and c is the greatest?

$$a = 2^{300}, b = 3^{200}, c = 6^{100}$$

Problem 7. Calculate $x \times y$, if

$$10y - y^2 = x^2 - 6x + 34.$$

Problem 8. Calculate

$$\frac{1}{2} + \left(\frac{1}{3} + \frac{2}{3}\right) + \left(\frac{1}{4} + \frac{2}{4} + \frac{3}{4}\right) + \left(\frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \frac{4}{5}\right) + \left(\frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6}\right) + \left(\frac{1}{7} + \frac{2}{7} + \frac{3}{7} + \frac{4}{7} + \frac{5}{7} + \frac{6}{7}\right)$$

Problem 9. Calculate

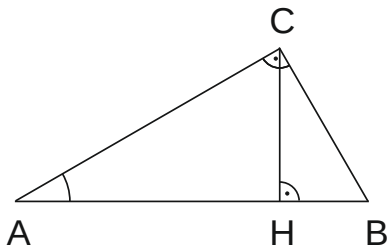
$$\sqrt{7-\sqrt{13}}-\sqrt{7+\sqrt{13}}$$

Problem 10. In how many ways can 4 children divide 6 identical apples between each other? (each child must get at least 1 apple)

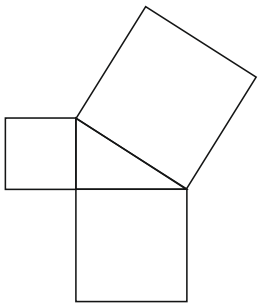
Hint: There are C_{N-1}^{n-1} natural numbers $x_1, x_2, \dots, x_n$, which are solutions to the equation

$$x_1 + x_2 + \cdots + x_n = N.$$

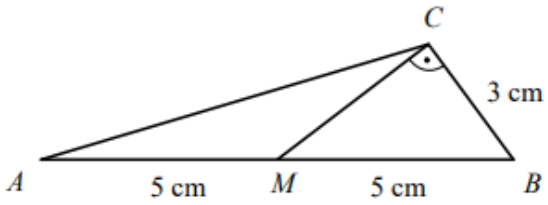
Problem 11. If $\measuredangle CAB = 30°$, $\measuredangle ACB = 90°$, $CH \perp AB, H \in AB$, calculate $AH \div AB$.



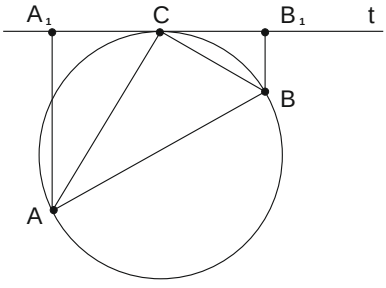
Problem 12. A square has been built on each side of a right-angled triangle with a hypotenuse of 4 cm. The resulting figure can be inscribed in a circle. Find the area of the triangle in cm^2.



Problem 13. If $AM = BM = 5\ cm$, $CB = 3\ cm$, $CM \perp CB$, calculate the area of ΔABC in cm^2.



Problem 14. The distances from the vertices A and B of ΔABC to the tangent t through the vertex C towards the circumscribed circle of this triangle are 9 cm and 4 cm, respectively. Calculate the altitude of ΔABC from the vertex C.



Problem 15. How many 5-digit numbers are there that are made up of different digits among 1, 2, 3, 4, 5 and 6, and are divisible by 3?

Problem 16. How many natural numbers n are there, for which

$$\sqrt{n^2 - 10n + 25} = 5 - n\,?$$

Problem 17. For which natural number N is the number $\sqrt{2} + \sqrt{3}$ a root of the equation

$$x^4 - N\,x^2 + 1 = 0?$$

Problem 18. Calculate x, if

$$\begin{cases} x \in (-1;2] \\ x^3 - [x] - 2 = 0 \end{cases}$$

Hint: $[x]$ is the greatest integer that does not exceed x.

Problem 19. Write down three digits after 2021, such that the resulting seven-digit number would be divisible by 7, 8 and 9. What number do you get? Write down all possible answers.

Problem 20. If

$$(1-2x) + (1-2x)^2 + (1-2x)^3 + (1-2x)^4 + (1-2x)^5 + (1-2x)^6 =$$
$$= a_6x^6 + a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$$

for the integers $a_0, a_1, \ldots, a_5, a_6$, calculate

$$\frac{a_1}{2} + \frac{a_2}{2^2} + \frac{a_3}{2^3} + \frac{a_4}{2^4} + \frac{a_5}{2^5} + \frac{a_6}{2^6}$$

WINTER – AGE GROUP 9

Problem 1. Calculate the value of the expression

$$\left(\sqrt{\left(1-\sqrt{2}\right)^2} + \sqrt{\left(\sqrt{2}-1\right)^2} + \sqrt[3]{8}\right)^2$$

Problem 2. Calculate the value of the expression

$$\frac{5 \times 14}{5 \times 8} + \frac{5 \times 14}{8 \times 11} + \frac{5 \times 14}{11 \times 14}$$

Problem 3. Find the natural number x, if $1 + 5 + 9 + \cdots + (4x-3) + (4x+1) = 91$

Problem 4. Calculate $4 \times x + 4 \times y - 5 \times z$, if $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$

Problem 5. The integer part $[x]$ of a number x is the greatest integer that does not exceed x. The fractional part $\{x\}$ of the number x is the number equal to $x - [x]$. Calculate

$$\left[\frac{\{1+\sqrt{2}\} - \{1-\sqrt{2}\}}{3}\right]$$

Problem 6. Calculate $f(-2)$, if

$$1 + x \times f(-2-x) = f(x)$$

Problem 7. Calculate x if $2x + 3y + 5z = -1.5$ and $3x + 6y + 10z = 5$.

Problem 8. Calculate x if

$$\sqrt{x^2} + \sqrt{x^2 - 2x + 1} = -3x$$

Problem 9. If $y = x^2 + 8x + 7$, simplify the following expression.

$$(x+1) \times (x+3) \times (x+5) \times (x+7) - y(y+8) + 7$$

Problem 10. Find the smallest natural number n for which the following inequality is true.

$$(\sqrt{3}+1)^n + (\sqrt{3}-1)^n > 8$$

Problem 11. An N-gon ($N \geq 4$) has $(4 \times N - 9)$ diagonals. Find N.

Problem 12. In the isosceles ΔABC $(AC = BC = 8\ cm)$, $\measuredangle ABC = 75°$, $BD \perp AC, D \in AC$; $DE \perp BC, E \in BC$. Find the length of the segment CE in cm.

Problem 13. Calculate the acute angle of a rhombus whose side is the geometric mean of its diagonals.

Problem 14. A right-angled triangle with legs a and b and a hypotenuse c has an area of 10 cm^2. Find the area of a rectangle with sides equal to $(-a + b + c)$ and $(a - b + c)$.

Problem 15. If a, b and c are sides of a triangle and

$$\mathrm{a} = 10\ \mathrm{cm} \leq \mathrm{b} \leq \mathrm{c} = 13\ \mathrm{cm},$$

find the greatest possible area of this triangle in cm^2.

Problem 16. Calculate

$$\frac{3sin\alpha + 2cos\alpha}{2sin^3\alpha + 3cos^3\alpha}$$

if $tg\alpha = -3$.

Problem 17. Find the natural number x, if $x-1$ is divisible by 18 and $x+1$ is a factor of 222.

Problem 18. If

$$a_1 = 2, a_2 = 9, a_3 = 16, a_4 = 23, \ldots$$
$$b_1 = 1007, b_2 = 999, b_3 = 991, b_4 = 983, \ldots$$

and $a_x = b_x$, find x.

Problem 19. The equation

$$x^3 - 2(b+1)x^2 + (2b^2 + 6b + 5)x = 0$$

has three real roots and b is a real parameter. What is the greatecт root?

Problem 20. How many five-digit numbers with different digits are there if the first and second digits are even?

SPRING – AGE GROUP 9

Problem 1. Calculate the value of the expression

$$2^{12} - 2^{11} - 2^{10} - \cdots - 2^2 - 2^1 - 2^0$$

Problem 2. For how many prime numbers p is the following inequality true?

$$p^3 > 16 > p$$

Problem 3. Calculate the value of the expression

$$\left(\sqrt{(2-\sqrt{5})^2}\right) \times (2+\sqrt{5})$$

Problem 4. If $x = \frac{1}{1-\sqrt{2}}$, calculate

$$|x| + |x-1| + 2x + 1$$

Problem 5. If the ordered pair of numbers (n, m) is a solution of the system of equations

$$\begin{vmatrix} xy = 4 \\ x^2y + y^2x + x + y = 30 \end{vmatrix}$$

calculate

$$n^2 + m^2.$$

Problem 6. Calculate $x \times y \times z$, if

$$\begin{vmatrix} 3^x = 5 \\ 5^y = 7 \\ 7^z = 81. \end{vmatrix}$$

Problem 7. Calculate $A - B$, if

$$\sqrt{A^2 + 6A + 13} + \sqrt{B^2 - 1} = 2.$$

Problem 8. Calculate

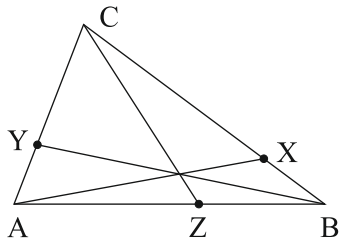
$$sin^2 15° + sin^2 25° + sin^2 35° + sin^2 45° + sin^2 55° + sin^2 65° + sin^2 75° + sin^2 90°$$

Problem 9. Find the smallest natural number x for which the following inequality is true.

$$(x - 5)(x^2 - 4x + 4) \geq 0$$

Problem 10. For which natural numbers n are the values of both $\frac{6}{n-1}$ and $\frac{14}{2n-1}$ natural numbers?

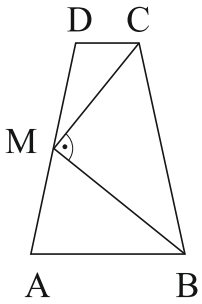
Problem 11. The points X, Y and Z lie on the sides BC, CA and AB of ΔABC.



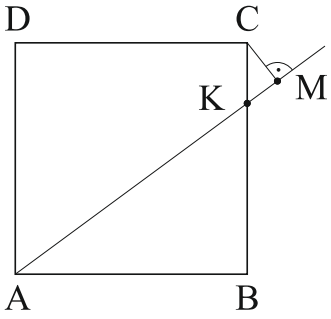
If the straight lines AX, BY and CZ intersect at one point and $\frac{AY}{YC} = \frac{1}{2}$, $\frac{BX}{XC} = \frac{1}{3}$, calculate

$$\frac{A_{\Delta AZC}}{A_{\Delta ABC}}$$

Problem 12. In an isosceles trapezium $ABCD$ with a leg BC = 2 cm, the point M is the midpoint of the leg AD and $\measuredangle CMB = 90°$. Find the perimeter of the trapezium.



Problem 13. A square $ABCD$ has sides lengths of 4 cm. The point K lies on the side BC and CK = 1 cm. Calculate the distance from the point C to the straight line AK in cm.



Problem 14. Numbers equal to 8^{16} and 625^{12} are written next to each other. How many digits are written?

Problem 15. How many four-digit numbers are written with at least one even digit?

Problem 16. If $tan\alpha + tan\beta = 2$ and $cot\alpha + cot\beta = 3$, find $tan(\alpha + \beta)$.

Problem 17. When dividing the two-digit number $\overline{ab}$ by 9, the remainder is 1. When dividing the same number by 10, the remainder is 2. Find the number $\overline{ab}$.

Problem 18. Calculate

$$\frac{a}{a+2} + \frac{4a^{-1}}{4a^{-1}+2}$$

Problem 19. If $a^2 + b^2 = 6ab$, calculate

$$\frac{4ab\sqrt{2}}{|a^2 - b^2|}.$$

Problem 20. If x_1 and x_2 are roots of the equation $x^2 - 2x - 2022 = 0$, calculate $2x_1 + x_2^2$.

FINAL – AGE GROUP 9

Problem 1. Find *N* if

$$\sqrt[4]{2 \times 2^{98} + 7 \times 2^{99}} \times \sqrt{2} = 2^N$$

Problem 2. If *a* and *b* are roots of the equation $x^2 + x - 999 = 0$, calculate

$$a^2 + 3b^2 + 2ab + 2b$$

Problem 3. Find the remainder left after dividing the natural number *N* by 5, if the number equal to

$$\frac{29 + 4.N^2}{9}$$

is a perfect square.

Problem 4. Find the sum of the real numbers x, y and z, if

$$\begin{cases} x + y = 2 \\ 2xy - z^2 = 2 \end{cases}$$

Problem 5. Calculate the product of the reciprocal values of x and y, if the following equation is true:

$$(x^2 - x + 1)(3y^2 - 2y + 3) = 2$$

Problem 6. Find y, if $a - x \neq 0$ and

$$x^3 - (a + 1) \times x^2 + a^2 = (a - x^2 + x + y) \times (a - x)$$

Problem 7. Find the greatest value of the natural number N for which exactly 8% of the numbers from 1 to N are divisible by 11.

Problem 8. Find the integer values of n, for which the value of the expression

$$\frac{n^2 + 4n - 3}{n + 2}$$

is an integer.

Problem 9. Find the smallest 3-digit number $\overline{abc}$ if the six-digit number $\overline{abcabc}$ has exactly 16 natural factors.

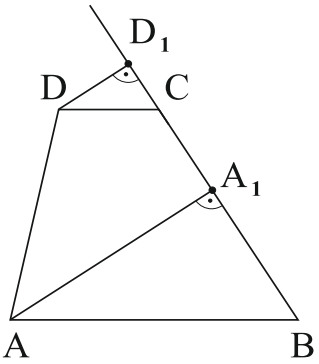
Problem 10. Find all numbers $x \in (0; 2\pi)$, for which $\sqrt{sinx + 5 - 4\sqrt{sinx + 1}} = 2 - \frac{\sqrt{6}}{2}$

Problem 11. How many different pyramids are there with all of their edges equal to 1 cm?

Problem 12. The greatest side of a triangle is $c = 2\ cm$, and the smallest side is $a = \sqrt{2}\ cm$. Find the greatest value of the area of such a triangle.

Problem 13. Let O be the centre of a circle circumscribed around ΔABC. Find the degree measure of $\measuredangle AOB$ if the quadrangle $AOBC$ is a rhombus.

Problem 14. The quadrangle $ABCD$ is a trapezium with an area of $25\ \text{cm}^2$ ($AB \parallel CD$, $AB > CD$), $AA_1 \perp BC, A_1 \in BC$, $DD_1 \perp BC, D_1 \in BC, BC = 5$ cm, $AA_1 = 7$ cm.
Find the length of the line segment DD_1 in cm.



Problem 15. Find the minimal number of chords in a given circle, such that the number of their intersecting points, which are inside the circle and are different from each other, is exactly 20.

Problem 16. Calculate the product of the integers x, y and z if all three conditions are satisfied:

$$\begin{cases} x^2 + 2x \le y \\ y^2 + 2y \le z \\ z^2 + 2z \le x \end{cases}$$

Problem 17. Find all integers *n* for which

$$|n^2 - 12n - 13|$$

is a prime.

Problem 18. We have 4 types of flowers. In how many ways can we make bouquets of 5 flowers each? The flowers of the same type are not different (they are considered the same).

Problem 19. How many of all the products of two different natural two-digit numbers are divisible by 5?

Problem 20. Calculate

$$\left(1+\frac{1}{1\times 3}\right)\times\left(1+\frac{1}{2\times 4}\right)\times\left(1+\frac{1}{3\times 5}\right)\times \ldots.\times\left(1+\frac{1}{n\times(n+2)}\right)\times\left(1+\frac{1}{(n+1)\times(n+3)}\right)$$

MATHEMATICAL RELAY – AGE GROUP 9

*The answer for each problem is represented by the symbols @, #, &, §, * and must be used to solve each following problem. Each team must fill one answer sheet.*

Problem 1. The hypotenuse of a right triangle is 80 cm, and its other two sides are integer centimetres. The smallest side of this triangle has a length of @ *cm*. Find @.

Problem 2. If

$A = \underbrace{44\ldots4}_{2\times @\text{ digits } 4}$, $B = \underbrace{11\ldots1}_{@+1 \text{ digits } 1}$, $C = \underbrace{66\ldots6}_{@ \text{ digits } 6}$, $D = \sqrt{A + B - C}$,

the number $3 \times D$ is written using # digits. Find #.

Problem 3. We are given the trapezium $ABCD$ ($AB \parallel CD$). A line segment has been drawn, which has endpoints on the legs of the trapezium, is parallel to the bases, and divides the trapezium into two parts with equal areas.
If $AB^2 + CD^2 = (\# + 1)\ cm^2$, the length of this line segment is &. Find &.

Problem 4. The number of real roots of the equation $x^2 - \sqrt{\& - x} = \&$
is §. Find §.

Problem 5. The remainder left after dividing $\S^{2202}$ by 100 is *. Find *.

MATHEMATICAL RELAY – AGE GROUP 9

ANSWER KEY

Problem	**1**	**2**	**3**	**4**	**5**
Answer	48	49	5	2	4

ANSWER KEY – AGE GROUP 9

Round / Problem	AUTUMN	WINTER	SPRING	FINAL
1	6 or 4	8	1	26
2	$x^2 - x + 2$	3	5	1999
3	2	6	1	2
4	1	-3	2	2
5	9	-1	28	6
6	b	-1	4	0
7	15	-8	$-2; -4$	75
8	10.5	-1	4.5	$-9; -3;$ $-1; 5$
9	$-\sqrt{2}$	7	2	101
10	10	3	4	$\frac{\pi}{6}; \frac{5\pi}{6}$
11	3:4	9	$\frac{3}{5}$	3
12	4	6	6	$\frac{\sqrt{7}}{2}$
13	12	$30°$	0.8	$120°$
14	6	40	49	3
15	240	60	8375	7
16	5	$\frac{70}{51}$	6	0; - 1
17	10	1, 73	82	0, 12
18	$\sqrt[3]{3}$	68	1	56
19	2021040, 2021544	0	1	1449
20	-6	5376	2026	$\frac{2(n+2)}{n+3}$

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